This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Educ T 168.98.610

HARVARD UNIVERSITY

LIBRARY OF THE

Department of Education

COLLECTION OF TEXT-BOOKS
Contributed by the Publishers

TRANSFERRED
TO
HARVARD COLLEGE
LIBRARY

3 2044 097 047 542

27

ELEMENTS

OF

PLANE AND SPHERICAL TRIGONOMETRY

·The M Co·

ELEMENTS

OF

PLANE AND SPHERICAL

TRIGONOMETRY

BY

J. W. NICHOLSON, A.M., LL.D.

PROFESSOR OF MATHEMATICS, LOUISIANA STATE UNIVERSITY AND
AGRICULTURAL AND MECHANICAL COLLEGE

New York
THE MACMILLAN COMPANY
LONDON: MACMILLAN & CO., Ltd.
1898

All rights reserved

Educ T 168.98.610

Harvard College Library
Dec. 20, 1918.
Transferred from
Education Library.

COPYRIGHT, 1898,
BY THE MACMILLAN COMPANY.

Norwood Press
J. S. Cushing & Co. – Berwick & Smith
Norwood Mass. U.S.A.

PREFACE

THIS book is designed for the use of beginners in the study of Trigonometry.

In *scope*, it presents the common and most essential elements. The author's aim has been to meet the wants of those schools and colleges which, on account of the multiplicity of subjects demanding the attention of the student, or for other causes, have the time or desire to teach only the leading principles of Trigonometry, — principles which are essential to the student's progress in higher mathematics, and on which the practical applications of the subject mainly depend.

In *treatment*, the endeavor has been to prepare a well-graded text-book at once simple, practical, thorough, and as full in details as is consistent with the plan of the work. In deducing and exemplifying principles the author has kept steadily in view the mental training of the student, as well as his advancement in the art of trigonometric computations. Several new features will be found, notably the "*Trigonometric Circle*" (Art. 23), and the introduction of the terms *opposite*, *adjacent*, and *like functions* (Arts. 18, 20, 25), which it is thought will serve to crystallize the student's ideas of the functions and their relations.

A few of the answers to examples and problems are withheld from each exercise. It is thought that the omissions may make the student more attentive to the principles upon which the solutions are based, and the checks by which the results are verified. The answers to the required angles are usually given to the nearest minute, which is sufficiently accurate for all ordinary practical purposes, and prevents the waste of time and labor in making closer determinations.

The logarithmic and trigonometric tables arranged specially for this work are based on the five-figure tables of Gauss, which have been modified to secure the convenience of compactness.

In preparing the book the author has consulted the best authorities both American and European. Some of the examples he has taken from these sources.

To his colleagues, Professors W. D. Taylor and T. W. Atkinson, who have rendered much assistance in reading proof-sheets, he wishes to express his grateful acknowledgments. With their help he has hoped to eliminate all accidental errors; if any have been overlooked, he asks that those finding them will have the kindness to send him word.

Baton Rouge, La.
April 18, 1898.

CONTENTS

PART I. PLANE TRIGONOMETRY

CHAPTER I

ACUTE ANGLES

PAGE
MEASUREMENT 1
TRIGONOMETRIC FUNCTIONS 3
FUNDAMENTAL FORMULAS 9
FUNCTIONS OF CERTAIN ANGLES 13

CHAPTER II

RIGHT TRIANGLES

NATURAL FUNCTIONS 17
LOGARITHMS 19
SOLUTION OF RIGHT TRIANGLES 20

CHAPTER III

ANGLES IN GENERAL

DIRECTED LINES AND ANGLES 32
FUNCTIONS OF ANGLES IN GENERAL 35
CIRCULAR OR RADIAN MEASURE OF ANGLES 40
LIMITING VALUES OF $\frac{\sin u}{u}$ AND $\frac{\tan u}{u}$ 42
REDUCTION OF FUNCTIONS 43
FUNCTIONS OF SUMS AND SUMS OF FUNCTIONS 48

CHAPTER IV

OBLIQUE TRIANGLES

RELATIONS OF SIDES AND ANGLES 56
SOLUTION OF OBLIQUE TRIANGLES 60
AREA OF AN OBLIQUE TRIANGLE 69

PART II. SPHERICAL TRIGONOMETRY

CHAPTER V

RIGHT SPHERICAL TRIANGLES

PAGE

DEFINITIONS AND ELEMENTARY PRINCIPLES 73
ELEMENTARY FORMULAS 74
SOLUTION OF RIGHT SPHERICAL TRIANGLES 77

CHAPTER VI

OBLIQUE TRIANGLES

RELATIONS OF SIDES AND ANGLES 83
SOLUTION OF OBLIQUE TRIANGLES 88
AREA OF SPHERICAL TRIANGLES 97
APPLICATIONS OF SPHERICAL TRIGONOMETRY 98

PART ONE

PLANE TRIGONOMETRY

CHAPTER I

ACUTE ANGLES

MEASUREMENT

1. Trigonometry treats of the measurement and properties of angles and triangles.

Plane figures only are considered in Plane Trigonometry.

2. To **measure** a quantity is to find how many times it contains a known quantity of the same kind, called the **Unit of Measure.**

The object of measuring a quantity is to determine its *size* or *value*. Trigonometry treats largely of measuring, or finding the sizes of, angles and lines by means of the relations they sustain to other known angles and lines.

3. The **Size or Value** of a quantity is expressed by prefixing to the name of the unit of measure the number which shows how many times that unit is contained in the quantity. As, 8 in., 10 ft.

4. In **Degree-Measure** of angles the unit of measure is $\frac{1}{90}$ of a right angle called a **degree** ($^\circ$); the degree is divided into 60 equal parts called **minutes** ($'$), and the minute is divided into 60 equal parts called **seconds** ($''$).

Thus, the angle which is $\frac{1}{16}$ of a right angle is written $\frac{90}{16}^\circ$, $5\frac{5}{8}^\circ$ or 5.625°, $5^\circ\ 37\frac{1}{2}'$ or $5^\circ\ 37.5'$, or $5^\circ\ 37'\ 30''$.

5. The **Relative Values** of two or more quantities are the numbers which show how many times they contain a common measure.

Thus, if the value of the line AB is m inches, and that of the line CD is n inches, the relative values of AB and CD are the pure numbers m and n, or $\frac{m}{n}$ and 1, CD being the unit of measure.

In general, mathematics treats only of the relative values of quantities, but in speaking of them the term "relative" is usually omitted. Thus, we say the values of the lines AB and CD are m and n.

Hence, in general, when letters are employed to represent the magnitudes of quantities, they stand for the relative, and not the concrete, values of the quantities; and, conversely, when lines are used to represent numbers, only the relative lengths of the lines are to be considered.

6. Two quantities are **commensurable** if there is a third quantity which is contained a whole number of times in each. The third quantity is called the **common measure.**

The ratio of any two commensurable quantities is equal to some simple common fraction, proper or improper.

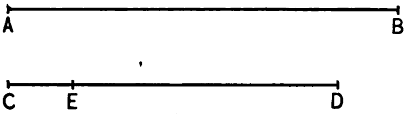

Fig. 1.

For, let the lines AB and CD, Fig. 1, represent the quantities, and suppose the common measure, say CE, to be contained in AB m times and in CD n times, then

$$AB = m \times CE \text{ and } CD = n \times CE.$$

$$\therefore \frac{AB}{CD} = \frac{m}{n}, \text{ where } m \text{ and } n \text{ are integers.}$$

Regarding CD as the unit, the value of AB is $\frac{m}{n}$, — a simple fraction showing the part which AB is of CD.

7. Two quantities are **incommensurable** if there is no third quantity which is contained a whole number of times in each.

There is no simple fraction equal to the ratio of two incommensurable quantities, but by taking the terms of the fraction sufficiently large there exists some simple fraction which comes as near to the ratio as we please.

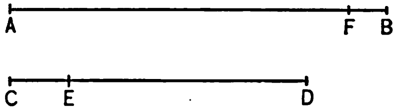

Fig. 2.

For, let the lines AB and CD, Fig. 2, represent the quantities. Divide CD into n parts, each equal to CE, say; then, $CD = n \times CE$.

Since AB and CD are incommensurable, CE is not contained in AB a whole number of times; let us suppose AB contains CE m times with a remainder FB less than CE. Then

$$AB = m \times CE + FB.$$

$$\therefore \frac{AB}{CD} = \frac{m \times CE + FB}{n \times CE} = \frac{m}{n} + \frac{1}{n} \times \frac{FB}{CE}.$$

Since $$\frac{FB}{CE} < 1, \quad \frac{1}{n} \times \frac{FB}{CE} < \frac{1}{n}.$$

$$\therefore \frac{AB}{CD} - \frac{m}{n} < \frac{1}{n}.$$

That is, the difference between the ratio $AB : CD$ and the simple fraction $\frac{m}{n}$ is less than $\frac{1}{n}$. Therefore, the larger n is, the smaller is the difference between the ratio and the fraction; and since n may be as large as we please, the fraction may come as near to the ratio as we please.

If CD is considered the unit, the **approximate value** of AB is $\frac{m}{n}$; which evidently differs from the true value of AB by a fraction less than $\frac{1}{n}$. In Trigonometry the approximate values only of the required quantities can, in general, be found, but these may be determined to any desired degree of accuracy.

TRIGONOMETRIC FUNCTIONS

8. Let $XOE (= u)$, Fig. 3, be any angle.* From *any* point in *either* side, as P, draw PB perpendicular to the other side (produced if necessary), forming the right triangle OBP.

The *relative* lengths of the sides OB, BP, OP evidently depend on the size of the $\angle u$; that is, *any change in the value of u produces a change in the value of each of the six ratios:*

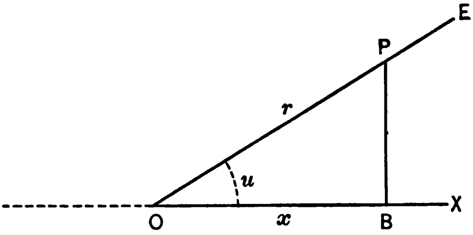

FIG. 3.

$$\frac{BP}{OP}, \quad \frac{OB}{OP}, \quad \frac{BP}{OB}, \quad \frac{OB}{BP}, \quad \frac{OP}{OB}, \quad \frac{OP}{BP}.$$

For this reason these **ratios** are called **Trigonometric Functions** † of the angle u.

---

* The angle u is supposed at present to be acute.

† For brevity, Trigonometric Functions are called simply Functions, when they are the only kind of functions considered.

9. Names and Notation. For convenience all these functions have been named. Representing the hypotenuse (*hype.*), OP, by r, the base or leg adjacent to u, OB, by x, and the perpendicular (*perp.*) or leg opposite to u, BP, by y, the names, values, and notation of the functions are as follows:

NAME.	FUNCTION.	NOTATION.
sine of u	$\frac{BP}{OP}$ or $\frac{\text{perp}}{\text{hype}}$.	$\sin u = \frac{y}{r}$.
cosine of u	$\frac{OB}{OP}$ or $\frac{\text{base}}{\text{hype}}$.	$\cos u = \frac{x}{r}$.
tangent of u . .	$\frac{BP}{OB}$ or $\frac{\text{perp}}{\text{base}}$.	$\tan u = \frac{y}{x}$.
cotangent of u . .	$\frac{OB}{BP}$ or $\frac{\text{base}}{\text{perp}}$.	$\cot u = \frac{x}{y}$.
secant of u	$\frac{OP}{OB}$ or $\frac{\text{hype}}{\text{base}}$.	$\sec u = \frac{r}{x}$.
cosecant of u . .	$\frac{OP}{BP}$ or $\frac{\text{hype}}{\text{perp}}$.	$\csc u = \frac{r}{y}$.

These names and values should be committed thoroughly to memory. The expressions $\sin u = \frac{y}{r}$, $\cos u = \frac{x}{r}$, etc., may be read:

$\sin u =$ *the perpendicular divided by the hypotenuse,*

$\cos u =$ *the base divided by the hypotenuse,*

$\tan u =$ *the perpendicular divided by the base,*

$\cot u =$ *the base divided by the perpendicular,*

$\sec u =$ *the hypotenuse divided by the base,*

$\csc u =$ *the hypotenuse divided by the perpendicular;*

or thus:

$\sin u =$ *the ratio of the opposite leg to the hypotenuse,*

$\cos u =$ *the ratio of the adjacent leg to the hypotenuse,*

$\tan u =$ *the ratio of the opposite leg to the adjacent leg,*

etc. etc. etc.

10. It is important to observe that the functions of any particular angle have always the same values *wherever* the perpendicular connecting the two sides may be drawn.

For, let P, P', Fig. 4, be any two points in the side OE, and P'' any point in the side OX; draw PB, $P'B' \perp OX$ and $P''B'' \perp OE$. Then, since the $\triangle$ OBP, $OB'P'$, $OB''P''$ are similar, we have

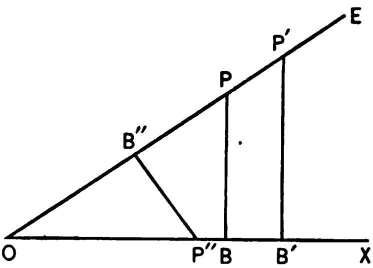

FIG. 4.

$$(1)\ \frac{BP}{OP}=\frac{B'P'}{OP'}=\frac{B''P''}{OP''};\ \therefore\ \sin u=\frac{BP}{OP},\ \text{or}\ \frac{B'P'}{OP'},\ \text{or}\ \frac{B''P''}{OP''};$$

$$(2)\ \frac{OB}{OP}=\frac{OB'}{OP'}=\frac{OB''}{OP''};\ \therefore\ \cos u=\frac{OB}{OP},\ \text{or}\ \frac{OB'}{OP'},\ \text{or}\ \frac{OB''}{OP''};$$

etc. etc.

Hence the functions depend only upon the *size* of the angle u. When their values are known, the angle, when acute, is known, precisely as if the number of degrees which it contains were known.

11. Versedsine and Coversedsine. Besides the six functions heretofore noticed there are two others called *versedsine* (*vers*) and *coversedsine* (*covers*), which are defined by the following equations:

$$\text{vers}\ u = 1 - \cos u, \qquad \text{covers}\ u = 1 - \sin u.$$

As these are of less importance than the others, and as they may be readily found from the cosine and sine, they are omitted in all that follows.

12. Inductive Exercises. 1. Which of the functions of u is $\frac{x}{y}$? *Ans.* cot. Which one is $\frac{x}{r}$? $\frac{y}{x}$? $\frac{y}{r}$? $\frac{r}{x}$? $\frac{r}{y}$?

2. How many of the functions involve y? How many x? r?

3. Which two do not involve x? *Ans.* sin, csc. Which two do not involve y? r?

4. Which two functions of u have y for a denominator? *Ans.* cot, csc. Which two have x? r?

5. Which two functions of u have r for a numerator? *Ans.* sec, csc. Which two have x? y?

6. Is $\frac{x}{r}$ the reciprocal of $\frac{r}{x}$? What does this show? *Ans.* That cos u is the reciprocal of sec u, and *vice versa*. What is the reciprocal of $\frac{y}{r}$? What does this show? Name two other functions that are the reciprocals of each other.

7. Which is the largest, x, y, or r? Why? Which is the larger, $\frac{y}{x}$ or $\frac{r}{x}$? What does this show? *Ans.* sec $u >$ tan u. Which is the larger, $\frac{x}{y}$ or $\frac{r}{y}$? What does this show?

8. Which is the smaller, cos u or cot u? Why? *Ans.* Because $\cos u = \frac{x}{r}$ and $\cot u = \frac{x}{y}$, and $\frac{x}{r} < \frac{x}{y}$. Which is the smaller, sin u or tan u? Why?

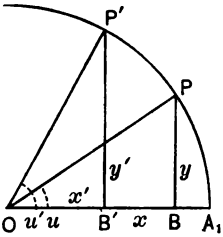

FIG. 5.

9. Which is the smaller, (1) sin u or 1? Why? (2) cos u or 1? Why? (3) sec u or 1? Why? (4) csc u or 1? Why?

Let us conceive the angle A_1OP' $(= u')$, Fig. 5, to be generated by the line $OP (= r)$ rotating about O, as a pivot, from $OA_1 (= r)$ to $OP' (= r)$. Draw $PB (= y)$ and $P'B' (= y') \perp OA_1$, and let $x = OB$, $x' = OB'$, and $u = A_1OP$.

Then, as OP rotates from OA_1 towards OP', the point P moves along the arc towards P', and the point B, keeping directly under P, moves towards B'. That is, as u approaches u', y approaches y', and x approaches x'.

10. As u increases or approaches u', will y increase or decrease? How about x? r? How about

$$\sin u \left(= \frac{y}{r}\right)? \quad \cos u \left(= \frac{x}{r}\right)? \quad \tan u \left(= \frac{y}{x}\right)?$$

$$\cot u \left(= \frac{x}{y}\right)? \quad \sec u \left(= \frac{r}{x}\right)? \quad \csc u \left(= \frac{r}{y}\right)?$$

Which three of the functions increase? Which three decrease?

11. When $u = 45°$, Fig. 6, are y and x equal? Why? *Ans.* When $u = 45°$, $v = 45°$, $\therefore x = y$. Are $\sin 45° \left(= \frac{y}{r}\right)$ and $\cos 45° \left(= \frac{x}{r}\right)$ equal? tan 45° and cot 45°? sec 45° and csc 45°? Show that $\tan 45° = 1$.

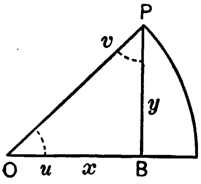

FIG. 6.

12. When $u < 45°$, which is the larger, x or y? Why? *Ans.* $u + v = 90°$, $\therefore$ when $u < 45°$, $v > 45°$; hence $v > u$, $\therefore x > y$. When $u < 45°$, which is the larger, sin u or cos u? tan u or cot u? sec u or csc u?

13. When $u > 45°$, which is the larger, x or y? Why? sin u or cos u? Why? tan u or cot u? Why? sec u or csc u? Why?

14. For any value of u between 0° and 90°, show that (1) sin u and cos u are proper fractions; (2) sec u and csc u are improper fractions; (3) tan u and cot u may be proper or improper fractions.

13. Inverse Trigonometric Functions. It is often convenient to designate an angle by the size of one of its functions. Thus, we may wish to express that a certain angle is one whose sine is $\frac{1}{2}$, which is done thus, $\sin^{-1}\frac{1}{2}$. This determines the angle (if acute) as much as to say, it is an angle of 30°. The expression $\sin^{-1}\frac{1}{2}$ is read "inverse sine $\frac{1}{2}$" or "anti-sine $\frac{1}{2}$," or "the angle whose sine is $\frac{1}{2}$."

Similarly, $\cos^{-1}x$ is read "inverse cosine x" or "anti-cosine x," or "the angle whose cosine is x." These and similar expressions, like $\tan^{-1}x$, $\sec^{-1}y$, etc., are the *inverse trigonometric functions.*

14. To construct an angle when one of its functions is given.

(1) Given $\sin u=\frac{2}{3}$ or $u=\sin^{-1}\frac{2}{3}$; construct u.

Since $\sin u=\frac{y}{r}$, $\frac{y}{r}=\frac{2}{3}$, hence (Art. 10), we construct a right △, Fig. 7, whose perp is 2 and whose hype is 3, and the ∠ at the base is u.

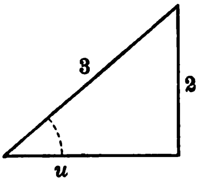

FIG. 7.

(2) Given $u=\sec^{-1}3$; construct u.

The secant 3 may be written $\frac{3}{1}$, and since $\sec u=\frac{r}{x}$, $\frac{r}{x}=\frac{3}{1}$, hence we construct a right △, Fig. 8, whose hype is 3 and whose base is 1, and the ∠ at the base is u.

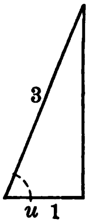

FIG. 8.

15. Exercise I.

Construct the following:

1. $\tan^{-1}\frac{3}{8}$; that is, an angle whose tangent is $\frac{3}{8}$.

2. $\cot^{-1}\frac{1}{4}$.
3. $\cos^{-1}\frac{1}{2}$.
4. $\tan^{-1}10$.
5. $\csc^{-1}5$.
6. $\sin^{-1}\frac{3}{11}$.
7. $\cos^{-1}\frac{4}{7}$.
8. $\cot^{-1}1$.
9. $\sec^{-1}\frac{8}{3}$.
10. $\csc^{-1}\frac{9}{4}$.
11. $\sin^{-1}\frac{3}{5}$.
12. $\cot^{-1}\frac{5}{8}$.
13. $\sec^{-1}\frac{7}{2}$.

Show that the following are impossible angles:

14. $\sin^{-1}\frac{3}{2}$.
15. $\csc^{-1}\frac{3}{5}$.
16. $\cos^{-1}2$.
17. $\sec^{-1}\frac{3}{4}$.
18. $\sin^{-1}\frac{1+n^2}{2n}$.
19. $\sec^{-1}\frac{1-n^2}{1+n^2}$.

16. To find the other functions of an angle when any one function is given.

(1) Given $\sin u = \frac{5}{13}$; find the other five functions of u.

Since $\sin u = \frac{y}{r}$, we make $y = 5$ and $r = 13$, and then find x thus:

$$x = \sqrt{r^2 - y^2} = \sqrt{169 - 25} = \sqrt{144} = 12.$$

Then, by the definitions,

$$\cos u = \frac{12}{13},\quad \tan u = \frac{5}{12},\quad \cot u = \frac{12}{5},\quad \sec u = \frac{13}{12},\quad \csc u = \frac{13}{5}.$$

(2) Given $\tan u = \frac{8}{15}$; find the other functions of u.

Since $\tan u = \frac{y}{x}$, we make $y = 8$, $x = 15$, and then find r thus:

$$r = \sqrt{x^2 + y^2} = \sqrt{225 + 64} = \sqrt{289} = 17.$$

$$\therefore\ \sin u = \frac{8}{17},\quad \cos u = \frac{15}{17},\quad \cot u = \frac{15}{8},\quad \sec u = \frac{17}{15},\quad \csc u = \frac{17}{8}.$$

(3) Given $\sec u = \sqrt{5}$; find the other functions of u.

Here we have $r = \sqrt{5}$, $x = 1$, $y = \sqrt{r^2 - x^2} = \sqrt{5 - 1} = 2.$

$$\therefore\ \sin u = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5},\quad \cos u = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5},\quad \tan u = 2,\ \text{etc.}$$

17. Exercise II.

In the following, find the remaining functions:

1. $\sin u = \frac{4}{5}$.
2. $\sin u = \frac{11}{61}$.
3. $\csc u = 7$.
4. $\tan u = \frac{7}{24}$.
5. $\tan u = \sqrt{3}$.
6. $\cos u = \frac{2}{\sqrt{13}}$.
7. $\sec u = \frac{41}{40}$.
8. $\sin 30° = \frac{1}{2}$.
9. $\cot 45° = 1$.
10. $\cos u = \frac{21}{29}$.
11. $\tan u = \frac{2n}{1 - n^2}$.
12. $\sin u = \frac{2ab}{a^2 + b^2}$.
13. Find $\cos \sin^{-1}\frac{9}{41}$; that is, the cos of an angle whose sin is $\frac{9}{41}$.

Since $\sin = \frac{y}{r}$, we make $y = 9$, $r = 41$, then $x = \sqrt{41^2 - 9^2} = 40$, and $\cos = \frac{x}{r} = \frac{40}{41}$, *Ans.*

In a similar manner find the following:

14. $\tan\cos^{-1}\frac{4}{5}$.

15. $\sec\cot^{-1}\frac{16}{63}$.

16. $\sin\tan^{-1}\frac{20}{21}$.

17. $\sin\sec^{-1}2$.

18. $\cot\csc^{-1}\frac{8}{3}$.

19. $\cos\tan^{-1}\sqrt{8}$.

20. $\sin\sin^{-1}a$.

21. $\tan\tan^{-1}a$.

22. $\csc\cot^{-1}\frac{2n}{1-n^2}$.

Ans. **1.** $\cos u=\frac{3}{5}$, $\tan u=\frac{4}{3}$, $\cot u=\frac{3}{4}$, $\sec u=\frac{5}{3}$, $\csc u=\frac{5}{4}$. **2.** $\cos u=\frac{60}{61}$, etc. **3.** $\sin u=\frac{1}{7}$, $\tan u=\frac{\sqrt{3}}{12}$, $\cot u=4\sqrt{3}$, $\cos u=\frac{4\sqrt{3}}{7}$, $\sec u=\frac{7\sqrt{3}}{12}$. **5.** $\sin u=\frac{\sqrt{3}}{2}$, $\cos u=\frac{1}{2}$, etc. **6.** $\sin u=\frac{3\sqrt{13}}{13}$, etc. **11.** $\cot u=\frac{1-n^2}{2n}$, $\sin u=\frac{2n}{1+n^2}$, $\cos u=\frac{1-n^2}{1+n^2}$, $\sec u=\frac{1+n^2}{1-n^2}$, $\csc u=\frac{1+n^2}{2n}$. **14.** $\frac{3}{4}$. **15.** $\frac{65}{16}$. **18.** $\frac{\sqrt{55}}{3}$. **20.** a. **22.** $\frac{1+n^2}{1-n^2}$.

FUNDAMENTAL FORMULAS

18. Opposite Functions are any two functions of which the numerator of either is the denominator of the other.

The following are the opposite functions:

(1)	(2)	(3)
$\frac{y}{r}$ and $\frac{r}{y}$,	$\frac{y}{x}$ and $\frac{x}{y}$,	$\frac{x}{r}$ and $\frac{r}{x}$,
sin and csc.	tan and cot.	cos and sec.

19. Principles. I. *The product of two opposite functions is equal to* 1.

For, $\frac{y}{r}\times\frac{r}{y}=1$, $\frac{y}{x}\times\frac{x}{y}=1$, $\frac{x}{r}\times\frac{r}{x}=1$.

$\therefore$ **sin csc = 1** . . . [1], **tan cot = 1** . . . [2], **cos sec = 1** . . . [3]

II. Cor. *Any function is the reciprocal of its opposite function.*

$\sin=\frac{1}{\csc}$ . . . (*a*)

$\csc=\frac{1}{\sin}$ . . . (*b*)

$\tan=\frac{1}{\cot}$ . . . (*c*)

$\cot=\frac{1}{\tan}$ . . . (*d*)

$\cos=\frac{1}{\sec}$ . . . (*e*)

$\sec=\frac{1}{\cos}$ . . . (*f*)

20. Adjacent Functions are any two functions having the same numerator or denominator.

Any two of the following functions which stand next to each other are adjacent functions:

$$\frac{y}{r}, \frac{x}{r}, \frac{x}{y}, \frac{r}{y}, \frac{r}{x}, \frac{y}{x}, \frac{y}{r}, \frac{x}{r}, \text{ etc.}$$

or sin, cos, cot, csc, sec, tan, sin, cos, etc. . . . . (*a*)

Let it be noticed (1) that each function has two adjacent functions, the one *before* and the other *after* it, the three being *consecutive* functions; and (2) that the series repeats itself after the sixth term.

21. Principles. I. *Any function is equal to the product of its two adjacent functions.*

For, $\frac{x}{r}=\frac{y}{r}\times\frac{x}{y}$. $\frac{x}{y}=\frac{x}{r}\times\frac{r}{y}$, etc.

$\therefore$ **cos = sin cot** . . . [4] **sec = csc tan** . . . . [7]

cot = cos csc . . . [5] **tan = sec sin** . . . . [8]

csc = cot sec . . . [6] **sin = tan cos** . . . . [9]

II. Cor. *Of any three consecutive functions the first is equal to the second divided by the third.*

$\sin=\frac{\cos}{\cot}$ or $\frac{\tan}{\sec}$ . . (*a*) $\sec=\frac{\tan}{\sin}$ or $\frac{\csc}{\cot}$ . . (*d*)

$\csc=\frac{\sec}{\tan}$ or $\frac{\cot}{\cos}$ . . (*b*) $\cot=\frac{\csc}{\sec}$ or $\frac{\cos}{\sin}$ . . (*e*)

$\cos=\frac{\cot}{\csc}$ or $\frac{\sin}{\tan}$ . . (*c*) $\tan=\frac{\sin}{\cos}$ or $\frac{\sec}{\csc}$ . . (*f*)

22. Since r, y, x, may be written $\frac{r}{1}, \frac{y}{1}, \frac{x}{1}$, the following may be treated as adjacent functions:

(1) $\frac{y}{x}, \frac{y}{1}, \frac{x}{1}, \frac{x}{y}$, or $\tan u$, y, x, $\cot u$ . . . . . (*a*)

(2) $\frac{x}{r}, \frac{x}{1}, \frac{r}{1}, \frac{r}{x}$, or $\cos u$, x, r, $\sec u$ . . . . . (*b*)

(3) $\frac{y}{r}, \frac{y}{1}, \frac{r}{1}, \frac{r}{y}$, or $\sin u$, y, r, $\csc u$ . . . . . (*c*)

$\therefore y=x\tan u$ . . . (*d*) $r=x\sec u$ . . . . (*g*)

$x=y\cot u$ . . . (*e*) $y=r\sin u$ . . . . (*h*)

$x=r\cos u$ . . . (*f*) $r=y\csc u$ . . . . (*j*)

These formulas may be derived thus:

$$\frac{y}{x}=\tan u,\ \therefore\ y=x\tan u;\quad \frac{x}{y}=\cot u,\ \therefore\ x=y\cot u;\ \text{etc.}$$

23. The Trigonometric Circle.* This is a mechanical device by which all the preceding formulas may be readily produced and easily memorized.

Describe a circle and complete the figure as shown below. Let the six functions of u and the three sides of the right triangle (x, y, r) be called *parts*, making *nine parts* in all.

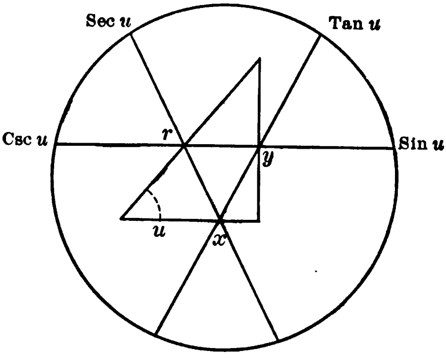

FIG. 8 *a*.

Now, let it be noticed that (1) the functions are placed around the circle in the order of (*a*), Art. 20; (2) the functions at the extremities of any chord are opposite functions, and (3) the parts along the chords are, respectively, the same as (*a*), (*b*), (*c*), Art. 22. That is, the opposite functions (Art. 18) are those which stand *opposite* to each other, and the adjacent functions (Art. 20) are those which stand *adjacent* to each other, whether taken along an arc or chord. Hence we have

I. *Any part is equal to the product of its adjacent parts.*

As $\tan u=\sec u\sin u,\quad y=x\tan u,\quad x=r\cos u$, etc.

II. *Of any three consecutive parts, the first is equal to the second divided by the third.*

As $\sec u=\frac{\csc u}{\cot u},\quad \sin u=\frac{\cos u}{\cot u},\quad y=\frac{r}{\csc u}$, etc.

III. *Any part is equal to the reciprocal of its opposite part.*

As $\sin u=\frac{1}{\csc u},\quad \cot u=\frac{1}{\tan u},\quad \sec u=\frac{1}{\cos u}$.

---

[*] This mnemonic device was invented by the author in 1868, and has been used by him in class work ever since.

24. Oral Exercise. These questions are intended to be answered by means of the Trigonometric Circle. The student should commit the circle and the rules to memory.

1. Express cos (1) in terms of sin and cot (by I, cos = sin × cot); (2) in terms of sin and tan (by II, cos = sin ÷ tan); (3) in terms of cot and csc; (4) of base (or x) and hype (or r); (5) of sec (by III, cos = 1 ÷ sec).

2. Express csc in terms of (1) sec and tan; (2) cot and cos; (3) hype and perp; (4) sec and cot; (5) sin.

3. In a similar manner express the value (1) of tan in five different ways; (2) of sin; (3) sec; (4) cot.

4. What is the value of y or perp in terms (1) of x or base and tan u (use I); (2) of r and sin u; (3) of x and cot u (use II); (4) of r and csc u.

5. In a similar manner express the value (1) of y in four different ways; (2) of x.

6. What function is the same as

(1) tan cos? (2) tan csc? (3) sec sin? (4) sec cot? (5) sin cot?

(6) $\frac{\tan}{\sec}$? (7) $\frac{\cot}{\cos}$? (8) $\frac{\csc}{\sec}$? (9) $\frac{\cos}{\sin}$? (10) $\frac{\tan}{\sin}$?

(11) $\frac{\cot}{\csc}$? (12) $\frac{1}{\tan}$? (13) $\frac{1}{\cos}$? (14) $\frac{1}{\csc}$? (15) $\frac{1}{\cos\csc}$?

25. Like Functions are those which have the same denominator. The like functions are

(1)	(2)	(3)
$\frac{y}{r}$ and $\frac{x}{r}$,	$\frac{y}{x}$ and $\frac{r}{x}$,	$\frac{x}{y}$ and $\frac{r}{y}$,
sin and cos.	tan and sec.	cot and csc.

26. Principle. *Any two like functions and* 1 *are relatively the three sides of a right triangle.*

For, by Geometry,

$$y^2 + x^2 = r^2 \qquad [10]$$

[10] $\div r^2$, $$\frac{y^2}{r^2} + \frac{x^2}{r^2} = 1, \text{ or } \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = 1.$$

[10] $\div x^2$, $$\frac{y^2}{x^2} + 1 = \frac{r^2}{x^2}, \text{ or } \left(\frac{y}{x}\right)^2 + 1 = \left(\frac{r}{x}\right)^2.$$

[10] $\div y^2$, $$1 + \frac{x^2}{y^2} = \frac{r^2}{y^2}, \text{ or } 1 + \left(\frac{x}{y}\right)^2 = \left(\frac{r}{y}\right)^2.$$

Substituting sin u for $\frac{y}{r}$, cos u for $\frac{x}{r}$, etc., and denoting $(\sin u)^2$ by $\sin^2 u$, $(\cos u)^2$ by $\cos^2 u$, etc., we have

$$\boldsymbol{\sin^2 u + \cos^2 u = 1} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [11]$$

$$\boldsymbol{\tan^2 u + 1 = \sec^2 u} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [12]$$

$$\boldsymbol{1 + \cot^2 u = \csc^2 u} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [13]$$

From [11], $\sin u = \sqrt{1-\cos^2 u}$, $\cos u = \sqrt{1-\sin^2 u}$ . . . (*a*)

From [12], $\tan u = \sqrt{\sec^2 u - 1}$, $\sec u = \sqrt{\tan^2 u + 1}$ . . . (*b*)

From [13], $\cot u = \sqrt{\csc^2 u - 1}$, $\csc u = \sqrt{\cot^2 u + 1}$ . . . (*c*)

By (*a*), we can find the sin (or cos) when the cos (or sin) is known; by (*b*), we can find the tan (or sec) when the sec (or tan) is known; by (*c*), we can find the cot (or csc) when the csc (or cot) is known.

Thus, if $\sin u = \frac{1}{2}$, $\cos u = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3}$;

if $\sec u = \frac{5}{3}$, $\tan u = \sqrt{\left(\frac{5}{3}\right)^2 - 1} = \sqrt{\frac{25}{9} - 1} = \sqrt{\frac{16}{9}} = \frac{4}{3}$;

if $\cot u = \frac{2}{5}$, $\csc u = \sqrt{\left(\frac{2}{5}\right)^2 + 1} = \sqrt{\frac{4}{25} + 1} = \sqrt{\frac{29}{25}} = \frac{1}{5}\sqrt{29}$.

FUNCTIONS OF CERTAIN ANGLES

27. Functions of the Complement of an Angle. In Fig. 3 let $v = OPB$, then v is the complement of u; that is, $u + v = 90°$, or $v = 90° - u$. The functions (1) sin and cos, (2) tan and cot, (3) sec and csc are called **Co-Functions.**

With respect to v, OB or x is the perpendicular or opposite leg and PB is the base or adjacent leg. Hence, by Art. 9,

$$\sin v = \frac{OB}{OP} (= \cos u), \qquad \cot v = \frac{BP}{OB} (= \tan u),$$

$$\cos v = \frac{BP}{OP} (= \sin u), \qquad \sec v = \frac{OP}{BP} (= \csc u),$$

$$\tan v = \frac{OB}{BP} (= \cot u), \qquad \csc v = \frac{OP}{OB} (= \sec u).$$

$\therefore$ $\sin(90° - u) = \cos u$ . . (*a*) $\qquad$ $\cot(90° - u) = \tan u$ . . (*d*)

$\cos(90° - u) = \sin u$ . . (*b*) $\qquad$ $\sec(90° - u) = \csc u$ . . (*e*)

$\tan(90° - u) = \cot u$ . . (*c*) $\qquad$ $\csc(90° - u) = \sec u$ . . (*f*)

Therefore,

I. *Any function of an acute angle is equal to the co-function of the complementary angle.*

II. *If a function of one acute angle is equal to the co-function of another acute angle, the sum of the two angles is 90°.*

Thus, by I, $\sin 70° = \cos 20°$, $\cot 85° 10' = \tan 4° 50'$;

by II, if $\sin A = \cos B$, or $\csc A = \sec B$, $A + B = 90°$.

28. Functions of 45°.

In Fig. 9, let $BOP = 45°$. Take $OB = 1$, and draw $BP \perp OB$. Then $v = 45°$, and therefore $BP = OB = 1$, and $OP = \sqrt{\overline{OB}^2 + \overline{BP}^2} = \sqrt{1+1} = \sqrt{2}$. Then, Art. 9,

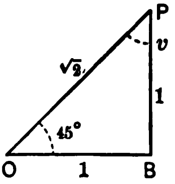

FIG. 9.

$$\sin 45° = \frac{1}{\sqrt{2}} \text{ or } \tfrac{1}{2}\sqrt{2}; \qquad \cot 45° = \frac{1}{1} \text{ or } 1;$$

$$\cos 45° = \frac{1}{\sqrt{2}} \text{ or } \tfrac{1}{2}\sqrt{2}; \qquad \sec 45° = \frac{\sqrt{2}}{1} \text{ or } \sqrt{2};$$

$$\tan 45° = \frac{1}{1} \text{ or } 1; \qquad \csc 45° = \frac{\sqrt{2}}{1} \text{ or } \sqrt{2}.$$

29. Functions of 30° and 60°.

Let ABD, Fig. 10, be an equilateral △ having each side equal to 2. Each angle will be 60°.

Draw $AC \perp BD$, then $BC = \tfrac{1}{2} BD = 1$, and $\angle CAB = 30°$; also, $AC = \sqrt{\overline{AB}^2 - \overline{BC}^2} = \sqrt{4-1} = \sqrt{3}$. Then, by Art. 9,

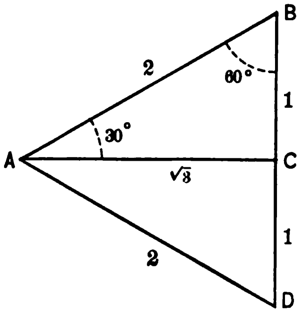

FIG. 10.

$$\sin 30° \text{ or } \cos 60° = \frac{1}{2};$$

$$\cos 30° \text{ or } \sin 60° = \frac{\sqrt{3}}{2};$$

$$\tan 30° \text{ or } \cot 60° = \frac{1}{\sqrt{3}} \text{ or } \tfrac{1}{3}\sqrt{3};$$

$$\cot 30° \text{ or } \tan 60° = \sqrt{3};$$

$$\sec 30° \text{ or } \csc 60° = \frac{2}{\sqrt{3}} \text{ or } \tfrac{2}{3}\sqrt{3};$$

$$\csc 30° \text{ or } \sec 60° = 2.$$

MISCELLANEOUS EXERCISES

30. Exercise III.

1. Prove $\tan u \cos u \csc u = 1$.

NOTE. — It is sufficient to show that the two members are the same.

1st *Proof.* Substitute $\frac{y}{x}$ for $\tan u$, $\frac{x}{r}$ for $\cos u$, $\frac{r}{y}$ for $\csc u$,

$$\frac{y}{x} \times \frac{x}{r} \times \frac{r}{y} = 1, \text{ or } 1 = 1.$$

2d *Proof.* By [9], $\tan u \cos u = \sin u$, and by [1], $\sin u \csc u = 1$.

In like manner prove the following:

2. $\sin u \cot u \sec u = 1$.
3. $\sec^2 u + \csc^2 u = (\sec u \csc u)^2$.

1st *Proof.* $\frac{r^2}{x^2} + \frac{r^2}{y^2} = \frac{r^4}{x^2y^2}$, or $r^2y^2 + r^2x^2 = r^4$, or $y^2 + x^2 = r^2$, or $r^2 = r^2$.

2d *Proof.* $[12] \times \csc^2 u$, $\csc^2 u + \tan^2 u \csc^2 u = \sec^2 u \csc^2 u$;

and by [7], $\csc^2 u + \sec^2 u = (\sec u \csc u)^2$.

4. $\cot^2 u = \cos^2 u + (\cot u \cos u)^2$.
5. $\tan^2 u = \sin^2 u + (\tan u \sin u)^2$.
6. $\tan u \sin u \cos u + \sin u \cos u \cot u = 1$.
7. $\sec u \tan u \sin u + 1 = \csc u \sec u \tan u$.
8. $(\sec u + \csc u)(1 - \cot u) = (\sec u - \csc u)(1 + \cot u)$.
9. $\sin^2 u \tan u + \cos^2 u \cot u + 2 \sin u \cos u = \tan u + \cot u$.
10. Given $4 \sin u = 3 \cos u$; find u.

Divide by $4 \cos u$, $\frac{\sin u}{\cos u} = \frac{3}{4}$, or $\tan u = \frac{3}{4}$. $\therefore u = \tan^{-1} \frac{3}{4}$.

In like manner find u in the following:

11. $5 \sin u = 3 \tan u$.
12. $7 \tan u = 5 \sec u$.
13. $10 \cos u = 3 \cot u$.
14. $9 \cot u = 5 \csc u$.
15. $\frac{2}{3} \sec u = \frac{3}{4} \csc u$.
16. $\frac{4}{5} \sec u = \frac{9}{10} \tan u$.
17. Given $\sin(60° + u) = \cos(80° - 6u)$; find u.

By Art. 27, $60° + u + 80° - 6u = 90°$, whence $u = 10°$.

18. If $\tan u = \cot(75° + 4u)$, what is the value of u?
19. If $\csc(20° - 8u) = \sec(55° + 3u)$, what is the value of u?

20. If $\cot u = \tan 7u$, what is the value of u?

21. If $\sin 3u = \cos(6u - 30°)$, what is the value of u?

Express the following as functions of an angle less than 45°:

22. $\tan 72°$. **23.** $\sin 64° 18'$. **24.** $\cos 81° 34' 26''$.

25. Given $\sin u = \frac{s}{1}$; express the other functions in terms of s.

By Art. 16, we construct a right △ whose hype is 1, perp s, and base $\sqrt{1-s^2}$; then $\cos u = \sqrt{1-s^2}$, $\tan u = \frac{s}{\sqrt{1-s^2}}$, etc.

26. Given $\cos u = c$; express the other functions in terms of c.

27. Given $\tan u = t$; express the other functions in terms of t.

28. Given $\sec u = \sec$; express the other functions in terms of sec.

29. Given $\csc u = \csc$; express the other functions in terms of csc.

Reduce the first member to the second in the following equations:

30. $\tan u \sin u \cot^2 u = \cos u$.

$\tan u \sin u \cot^2 u = (\tan u \cot u)(\sin u \cot u) = \cos u$, by [2], [4].

31. $(\sin u + \cos u)(\tan u + \cot u) = \sec u + \csc u$.

$$(\sin u + \cos u)\left(\frac{\sin u}{\cos u} + \frac{\cos u}{\sin u}\right) = \frac{\sin^2 u}{\cos u} + \cos u + \sin u + \frac{\cos^2 u}{\sin u}$$

$$= \frac{(\sin^2 u + \cos^2 u)\sin u + (\sin^2 u + \cos^2 u)\cos u}{\sin u \cos u}$$

$$= \frac{\sin u + \cos u}{\sin u \cos u} \text{ (by [11])}, = \frac{1}{\cos u} + \frac{1}{\sin u}$$

$= \sec u + \csc u$, by (*b*), (*f*), Art. 19.

32. $\frac{1 - \cos u}{1 + \cos u} = (\csc u - \cot u)^2$. **33.** $\frac{\tan u + \tan v}{\cot u + \cot v} = \tan u \tan v$.

34. $(\sin u \cos v + \cos u \sin v)^2 + (\cos u \cos v - \sin u \sin v)^2 = 1$.

35. $(\sin u \cos v - \cos u \sin v)^2 + (\cos u \cos v + \sin u \sin v)^2 = 1$.

36. $(x \cos u - y \sin u)^2 + (x \sin u + y \cos u)^2 = x^2 + y^2$.

Answers: **11.** $\cos u = \frac{3}{5}$, or $u = \cos^{-1}\frac{3}{5}$. **13.** $u = \sin^{-1}\frac{3}{10}$. **15.** $u = \tan^{-1}\frac{9}{8}$. **18.** $u = 3°$. **20.** $u = 11\frac{1}{4}°$. **22.** $\cot 18°$. **24.** $\sin 8° 25' 34''$. **26.** $\sin u = \sqrt{1-c^2}$, $\tan u = \frac{\sqrt{1-c^2}}{c}$, $\cot u = \frac{c}{\sqrt{1-c^2}}$, $\sec u = \frac{1}{c}$, $\csc u = \frac{1}{\sqrt{1-c^2}}$. **28.** $\tan u = \sqrt{\sec^2 - 1}$, $\sin u = \frac{\sqrt{\sec^2 - 1}}{\sec}$, $\cos u = \frac{1}{\sec}$, $\cot u = \frac{1}{\sqrt{\sec^2 - 1}}$, $\csc u = \frac{\sec}{\sqrt{\sec^2 - 1}}$.

CHAPTER II

RIGHT TRIANGLES

NATURAL FUNCTIONS

31. The functions of angles are usually called **Natural Functions** to distinguish them from the logarithms of the functions, which are called **Logarithmic Functions.**

The natural functions of certain angles have been found, Arts. 28, 29, which, as a rule, are expressed by means of radicals, but the number of angles whose functions can be determined in this manner is comparatively very small.

In general, the sides *OP*, *OB*, and *BP*, Fig. 3, are incommensurable; and, therefore, there are no simple fractions which are exactly equal to the functions, but we may find fractions which come as near to the functions as we please (Art. 7).

32. Roughly, the approximate values of the functions of any given angle, as 34° 40′, may be found thus:

With a protractor lay off the angle $XOE = 34° 40'$. Take any distance $OP (= 10$ in., say), the greater the better, and draw $PB \perp OX$. Then measure carefully *OB* and *BP*, and it will be found that $OB = 8.2$ in. and $BP = 5.7$ in., nearly.

Therefore, roughly,

$$\sin 34° 40' = \frac{5.7}{10} = .57,$$

$$\cos 34° 40' = \frac{8.2}{10} = .82,$$

$$\tan 34° 40' = \frac{5.7}{8.2} = .69,$$

$$\cot 34° 40' = \frac{8.2}{5.7} = 1.44,$$

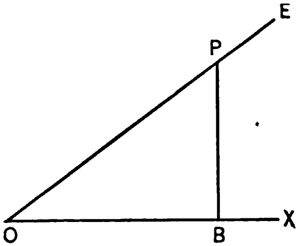

FIG. 11.

$$\sec 34° 40' = \frac{10}{8.2} = 1.22, \qquad \csc 34° 40' = \frac{10}{5.7} = 1.75.$$

In this manner let the student find the *sin*, *cos*, *tan*, and *cot* of 20°, and of 32° 30′, and compare the results with the following:

Angle	sin	cos	tan	cot
20°	.34202	.93969	.36397	2.7475
32° 30′	.53730	.84339	.63707	1.5697

33. Expression of Approximate Values. When the omitted part of the value is equal to or greater than .5 of the unit of the last figure retained, that figure must be increased by 1. Thus, 5.82637, expressed approximately by four places of decimals, is 5.8264; by three places, 5.826; by two places, 5.83; by one place, 5.8; and as an integer, 6.

Similarly, 12° 34′ 46″, expressed in the nearest *tenth* of a minute, is 12° 34.8′; in the nearest minute, 12° 35′; the nearest tenth of a degree, 12.6°; and in the nearest degree, 13°.

NOTE. — Throughout this work, the required angles will often be expressed in the nearest minute, as that is sufficiently accurate for all ordinary practical purposes, and spares the student the labor and waste of time in finding closer determinations.

34. A Table of Natural Functions, as arranged on pp. 59 to 61, is a table by which we may find, approximately,

(1) The natural sin, cos, tan, or cot of any acute angle;

(2) Any acute angle when its sin, cos, tan, or cot are known.

Directions as to the use of the table will be found in the introduction to the table.

35. Exercise IV.

Find the following:

1. sin 9° 30′.
2. cos 78° 40′.
3. tan 35° 16′.
4. cot 23° 25′.
5. sin 64° 15.6′.
6. cot 70° 37′.
7. tan 13° 43′.
8. cos 85° 22.8′.
9. sin 2° 49′.
10. Find the angle whose sin is $\frac{2}{3}$ or .66667.
11. Find the angle whose tan is .70238.
12. Find the angle whose cos is .38671.

Express the following in degree measure.

13. $\sin^{-1} .41204$.
14. $\tan^{-1} \frac{3}{4}$.
15. $\cot^{-1} \frac{125}{120}$.
16. $\cos^{-1} .82116$.
17. $\tan^{-1} \frac{10}{7}$.
18. $\cot^{-1} \frac{3.2}{4.5}$.

Answers: 1. .16505. 3. .70717. 5. .90078. 10. 41° 48.6′. 12. 67° 15′. 14. 36° 52.2′. 18. 54° 35′.

LOGARITHMS

36. Tables of Logarithms. A table of natural functions is sufficient for all the purposes of trigonometric computations, but it is more convenient to employ a table of logarithms. A table of the logarithms of numbers from 1 to 10,000 will be found on pp. 20 to 35, and a table of the logarithms of the sin, cos, tan, and cot of acute angles on pp. 36 to 58.

37. The properties of logarithms and the method of finding and applying them are fully explained and illustrated in the introduction to the tables; and if the student is not already familiar with the subject, he should read that part of the book before entering upon the solution of triangles.

38. "The sole arithmetical operations which can be performed by logarithms are those of multiplication, division, involution, and evolution. Before, therefore, the value of an expression can be calculated by means of logarithms, the expression must be put into such a form that no other arithmetical operations than these have to be performed. Such an arrangement of an expression is called the adaptation of it to logarithmic computation." — *Snowball.*

Thus, in the right $\triangle OBP$, Fig. 3, if r and y are known and x required, logarithms cannot be directly applied to determine the value of x from the expression $x = \sqrt{r^2 - y^2}$, but the expression is adapted to logarithmic computation by writing it $x = \sqrt{(r + y)(r - y)}$

39. Exercise V.

Find the logarithms of:

1. 425.
2. 0.617.
3. 2347.
4. 108457.
5. .30365.
6. 24.725.
7. .00065.
8. .0042683.
9. $(.034)^3$.
10. $(1.252)^4$.
11. $\sqrt{.75}$.
12. $\sqrt[3]{.03256}$.
13. $\sin 45° 28'$.
14. $\tan 16° 35'$.
15. $\cos 69° 14'$.
16. $\tan 45° 59'$.
17. $\cot 70° 18'$
18. $\sin 13° 12.4'$.

Find the numbers corresponding to the logarithms:

19. 2.47246.
20. 1.26784.
21. 9.79029 – 10.
22. 1.36435.
23. 4.85378.
24. 6.15465 – 10.

Find the angle u if:

25. $\log \sin u = 9.70075$.
26. $\log \cos u = 9.43758$.
27. $\log \tan u = 9.79840$.
28. $\log \cot u = 9.84266$.

Find by logarithms the value of:

29. 3.1425×603.93.

30. $25.073 \times .0375$.

31. $4867.2 \div 765.16$.

32. $45607 \div 31045$.

33. $\dfrac{.06547}{74.938 \times .05938}$.

34. $\dfrac{3.4 \times .0275 \times .76}{.00381 \times 43.07}$.

Answers: **3.** 3.37051. **4.** 5.03526. **5.** 9.48237 – 10. **8.** 7.63025 – 10. **9.** 5.59444 – 10. **11.** 9.93753 – 10. **13.** 9.85299 – 10. **15.** 9.54969 – 10. **16.** .01491. **19.** 296.8. **24.** .00014277. **25.** 30° 8.1′. **27.** 32° 9.3′. **29.** 1897.9. **31.** 6.361. **34.** .433.

SOLUTION OF RIGHT TRIANGLES

40. A triangle has six parts or elements, three sides and three angles. To **solve** a triangle is to find from its given parts the values of its unknown parts.

41. Any right triangle can be solved,

I. *If any one of the sides and either of the acute angles are given;*

II. *If any two of the sides are given.*

42. The *Complete Solution* of a triangle consists of two parts, viz.:

(1) The **Statement**, which consists in expressing the unknown in terms of the given parts by means of formulas.

(2) The **Computation**, which consists in applying the formulas to the given values and computing the values of the required parts.

43. Notation. The notation will remain as heretofore: u and v will denote the acute angles, y and x their opposite sides, respectively, and r the hypotenuse.

44. Suggestions in Reference to Solving Triangles in General. (1) Draw roughly the figure representing the triangle to be solved, and mark with a cross or otherwise the given parts, as in the following examples.

(2) After finding one of the required parts, there are generally two or more ways by which the remaining part or parts may be determined. Of these, select the method which involves the least labor, provided that it is equally free from liability to error. An error committed in finding a part will vitiate the remainder of the work if that part is involved in the subsequent formulas. Hence, the unknown quantities should be computed directly from the data, or from the parts whose values have been very carefully determined.

(3) For a check upon the accuracy of the computation, take one or more of the omitted formulas by which the required parts might have been found, and apply it to the parts in such a manner that any mistake in the computations of

the parts will make the check a false equation. Small errors disclosed by the check are often due to the inaccuracy of the tables, and not to mistakes in the computation.

(4) Solving triangles by logarithms is generally much less laborious than solving them by natural functions; yet the latter is the natural method, and in employing it the student is less liable to lose sight of the character and potentiality of the functions. It should therefore be employed sufficiently to insure its comprehension. It may be used as a check on results obtained by logarithms.

45. Inductive Exercises.* 1. If the hypotenuse (r) of a right triangle and the angle (u) at the base were known, (1) how would the perp (y) be found? *Ans.* By multiplying the hype by $\sin u$. (2) How would the base (x) be found?

2. If y and u were known, (1) how would r be found? *Ans.* By dividing y by $\sin u$. (2) How would you find x?

3. If x and u are known, how do you find (1) r? (2) y?

4. What is the value of y when $r = 20$ and $\sin u = \frac{3}{5}$?

5. What is the value of x when $r = 12$ and $\cos u = \frac{3}{4}$?

6. Find x and y when $r = 18$ and $u = 30°$. (For $\sin 30°$, see Art. 29.)

7. What is the value of r, (1) when $y = 12$ and $\sin u = \frac{3}{4}$? (2) When $x = 25$ and $\cos u = \frac{5}{6}$? (3) When $y = 10$ and $u = 30°$? (4) When $x = 12$ and $u = 60°$?

8. What is the value of y when $x = 16$ and $\tan u = \frac{3}{8}$? What is the value of x when $y = 30$ and $\tan u = \frac{5}{7}$? Find y when $x = 6$ and $u = 30°$. Find x when $y = 10$ and $u = 45°$.

9. How would you find u if y and r were known? *Ans.* I would find $\sin u$ by dividing y by r. How would you find u, (1) if x and r were known? (2) If x and y were known?

10. What is the value of u when $x = 7$ and $y = 7$? *Ans.* 45°, Art. 28.

11. What is the value of u when $x = 5$ and $r = 10$?

12. What is the value of u when $y = 6\sqrt{3}$ and $r = 12$?

13. Find u when $x = b$ and $y = p$. *Ans.* $u = \tan^{-1}\frac{p}{b}$.

14. Find u when $x = b$ and $r = h$.

15. Find u when $y = p$ and $r = h$.

16. Find u by the table, p. 59, (1) when $x = 3$, $r = 5$. (2) When $x = 5$, $y = 4$. (3) When $y = 7$, $r = 10$.

---

* These questions are intended to be answered orally by means of the Trigonometric Circle. If the student has not committed the Circle to memory, let him make a rough copy of it on a slip of paper, and hold it before him. The sec and csc are to be omitted, as they are not usually employed in practical computations.

46. I. **When a side and an acute angle are given.**

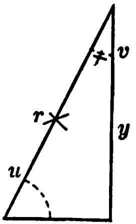

FIG. 12.

(1) Given $r = 20$, $v = 21° 47'$; find u, x, y.

STATEMENT

$$u + v = 90°, \therefore\ u = 90° - v.$$

$$\frac{y}{r} = \sin u, \qquad \therefore\ y = r \sin u.$$

$$\frac{x}{r} = \cos u, \qquad \therefore\ x = r \cos u \quad . \quad . \quad . \quad . \quad (a)$$

$$\textit{Check: } x^2 + y^2 = r^2, \ \therefore\ x = \sqrt{(r + y)(r - y)} \quad . \quad (b)$$

NOTE. — We may find x by (b) and check by (a).

COMPUTATION. (1) BY NATURAL FUNCTIONS

$$u = 90° - 21° 47' = 68° 13'.$$

$$y = 20 \times \sin 68° 13' = 20 \times .92859 = 18.572.$$

$$x = 20 \times \cos 68° 13' = 20 \times .37110 = 7.422.$$

(2) BY LOGARITHMS

$$\begin{aligned}
\log y &= \log r + \log \sin u. \\
\log r &= 1.30103 \\
\log \sin u &= \underline{9.96783 - 10} \\
\log y &= 1.26886 \\
y &= 18.572.
\end{aligned}$$

$$\begin{aligned}
\log x &= \log r + \log \cos u. \\
\log r &= 1.30103 \\
\log \cos u &= \underline{9.56949 - 10} \\
\log x &= .87052 \\
x &= 7.422.
\end{aligned}$$

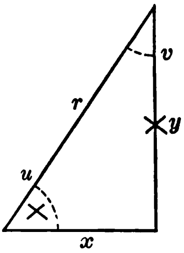

FIG. 13.

(2) Given $y = 78$, $u = 62° 10'$; find v, x, r.

STATEMENT

$$u + v = 90°, \qquad \therefore\ v = 90° - u.$$

$$\frac{x}{y} = \cot u, \qquad \therefore\ x = y \times \cot u.$$

$$\frac{y}{r} = \sin u, \qquad \therefore\ r = y \div \sin u.$$

COMPUTATION BY LOGARITHMS

$$\begin{aligned}
\log x &= \log y + \log \cot u. \\
\log y &= 1.89209 \\
\log \cot u &= \underline{9.72262 - 10} \\
\log x &= 1.61471 \\
\therefore\ x &= 41.182.
\end{aligned}$$

$$\begin{aligned}
\log r &= \log y + \text{colog} \sin u. \\
\log y &= 1.89209 \\
\text{colog} \sin u &= \underline{.05340} \\
\log r &= 1.94549 \\
\therefore\ r &= 88.204.
\end{aligned}$$

(3) Given $x = 98.42$, $u = 22° 59'$; find y, r.

STATEMENT

$\frac{y}{x} = \tan u$, $\quad \therefore\ y = x \tan u$. $\quad$ *Ans.* $y = 41.74$.

$\frac{x}{r} = \cos u$, $\quad \therefore\ r = x \div \cos u$. $\quad r = 106.9$.

Let the student make the computation by both methods.

47. Exercise VI.

1. Given $r = 125.7$, $v = 75° 12'$; find $x = 121.53$, $y = 32.11$.
2. Given $r = 325$, $u = 27° 34'$; find $x = 288.1$, $y = 150.4$.
3. Given $y = 35.8$, $u = 61° 13'$; find $r = 40.85$, $x = 19.67$.
4. Given $x = 67.916$, $v = 43° 48'$; find $r = 98.12$, $y = 70.82$.
5. Given $y = 48.532$, $v = 23° 16'$; find $r = 52.83$, $x = 20.87$.
6. Given $x = 76.56$, $u = 82° 6.3'$; find $r = 557.4$, $y = 552.1$.
7. Given $x = .13534$, $v = 14° 55.7'$; find $r = .5254$, $y = .5076$.
8. Given $r = 469.423$, $u = 26° 12.4'$; find $y = 207.3$, $x = 421.2$.
9. Given $y = .04804$, $u = 31° 45'$; find $r = .0913$, $x = .0776$.

Solve the following right triangles:

10. $r = 100$, $u = 30° 40'$.
11. $r = 34.372$, $u = 18° 16.5'$.
12. $x = 68.2$, $v = 16° 37'$.
13. $x = .07456$, $v = 52° 42.8'$.
14. $y = 8.57$, $u = 61° 48'$.
15. $y = .00824$, $u = 27° 34.3'$.
16. $r = .504$, $v = 12° 5.6'$.
17. $x = .00093$, $u = 41° 28.5'$.

48. II. When two sides are given.

(1) Given $y = 35$, $r = 62$; find u, x.

STATEMENT

$\sin u = \frac{y}{r}$, $\quad \therefore\ u = \sin^{-1}\frac{y}{r}$.

$\frac{x}{r} = \cos u$, $\quad \therefore\ x = r \cos u$.

Or $\quad x = \sqrt{(r + y)(r - y)}$.

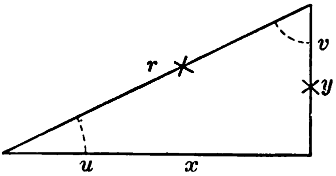

FIG. 14.

COMPUTATION. (1) BY NATURAL FUNCTIONS

$$u = \sin^{-1}\frac{35}{62} = \sin^{-1}.56452 = 34° 22.1'.$$

$$x = 62 \times \cos u = 62 \times .82543 = 51.176.$$

(2) BY LOGARITHMS

$$\begin{aligned} \log\sin u &= \log y + \operatorname{colog} r. \\ \log y &= 1.54407 \\ \operatorname{colog} r &= \underline{8.20761 - 10} \\ \log\sin u &= 9.75168 - 10 \\ u &= 34^\circ\ 22.1'. \end{aligned}$$

$$\begin{aligned} \log x &= \log r + \log\cos u. \\ \log r &= 1.79239 \\ \log\cos u &= \underline{9.91668 - 10} \\ \log x &= 1.70907 \\ x &= 51.176. \end{aligned}$$

(2) Given $y = 273$, $x = 418$; find u, r.

STATEMENT

$$\tan u = \frac{y}{x}, \qquad \therefore\ u = \tan^{-1}\frac{y}{x}.$$

$$\frac{y}{r} = \sin u, \qquad \therefore\ r = y \div \sin u.$$

COMPUTATION BY NATURAL FUNCTIONS

$$u = \tan^{-1}\frac{273}{418} = \tan^{-1}.65311 = 33^\circ\ 8.9'.$$

$$r = 273 \div \sin 33^\circ\ 8.9' = 273 \div .54681 = 499.25.$$

Let the student verify these results by logarithms.

(3) Given $x = 12$, $r = 13$; find u, v, y.

STATEMENT

$$y^2 + x^2 = r^2, \qquad \therefore\ y = \sqrt{r^2 - x^2}.$$

$$\tan u = \frac{y}{x}, \qquad \therefore\ u = \tan^{-1}\frac{y}{x}.$$

$$v = 90^\circ - u.$$

COMPUTATION

$$y = \sqrt{169 - 144} = \sqrt{25} = 5.$$

$$u = \tan^{-1}\frac{5}{12} = \tan^{-1}.41667 = 22^\circ\ 37.2'.$$

2D STATEMENT

$$\cos u = \frac{x}{r}, \qquad \frac{y}{r} = \sin u, \qquad \therefore\ y = r\sin u.$$

COMPUTATION BY LOGARITHMS

$$\begin{aligned} \log\cos u &= \log x + \operatorname{colog} r \\ \log x &= 1.07918 \\ \operatorname{colog} r &= \underline{8.88606 - 10} \\ \log\cos u &= 9.96524 \\ u &= 22^\circ\ 37.2'. \end{aligned}$$

$$\begin{aligned} \log y &= \log r + \log\sin u. \\ \log r &= 1.11394 \\ \log\sin u &= \underline{9.58503 - 10} \\ \log y &= .69897 \\ y &= 5. \end{aligned}$$

49. Exercise VII.

1. Given $x = 25.1$, $r = 32.7$; find $u = 39° 52'$, $y = 20.96$.
2. Given $y = 3.414$, $x = 2.875$; find $u = 49° 53.9'$, $r = 4.463$.
3. Given $y = 99.464$, $r = 156.82$; find $u = 39° 21.9'$, $x = 121.2$.
4. Given $y = 694.73$, $x = 8372.1$; find $u = 4° 44.6'$, $r = 8401.6$.
5. Given $r = 444.4$, $y = 150$; find $u = 19° 43.6'$, $x = 418.3$.
6. Given $r = 726.9$, $x = 316.2$; find $u = 64° 12.9'$, $y = 654.5$.
7. Given $r = .0941$, $y = .0536$; find $u = 34° 43.3'$, $x = .07734$.
8. Given $y = 2.1$, $x = 2$; find $u = 46° 23.8'$, $r = 2.9$.
9. Given $r = 1.84$, $x = 1.47$; find $u = 36° 58.4'$, $y = 1.107$.

Solve the following right triangles:

10. $r = 46$, $x = 32$.
11. $x = 342.5$, $y = 92.87$.
12. $r = 100$, $y = 57$.
13. $x = 4.3758$, $r = 10.0824$.
14. $x = 65$, $y = 72$.
15. $y = .06437$, $r = .08342$.
16. $r = .16$, $x = .09$.
17. $x = .00221$, $y = .00099$.
18. The hypotenuse and base of a right triangle are 308.07 and 151.24. Show that the perpendicular and the angle at the base are 268.39 and 60° 36′.
19. The length and width of a rectangle are 13 and 7. Find the angles into which the corner is divided by the diagonal.

50. When the hypotenuse and one leg are nearly equal. When the hypotenuse and one leg are given, and these given parts are nearly equal in value, the preceding method of solution will lack precision. For, let r and x be the given and nearly equal parts, then u must be nearly 0° and v nearly 90°, and to find u and v, we have

$$\cos u = \frac{x}{r}, \quad \sin v = \frac{x}{r}.$$

Now the differences between the cosines or log cosines of angles near 0°, and the sines or log sines of angles near 90°, are so small that they do not appear in the tables. Thus:

(1) $\log \cos 2° 13' \text{ or } \log \sin 87° 47' = 9.99967 - 10$,

(2) $\log \cos 2° 15' \text{ or } \log \sin 87° 45' = 9.99967 - 10$.

Therefore the angle whose

(1) log cos is $9.99967 - 10$, may be any angle between 2° 12′ and 2° 16′;

(2) log sin is $9.99967 - 10$, may be any angle between 87° 44′ and 87° 48′.

We will now deduce a formula by which more accurate results can be obtained in such cases.

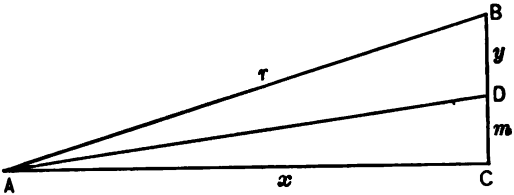

FIG. 15.

Let AD bisect the angle $CAB (= u)$, and let $m = CD$, then

$$\tan \tfrac{1}{2} u = \frac{m}{x}.$$

By Geometry, $r : x :: BD : m.$

By composition, $r + x : x :: BD + m : m,$

or $r + x : x :: y : m.$

Whence, $$\frac{r+x}{x} = \frac{y}{m}, \text{ or } \frac{m}{x} = \frac{y}{r+x}.$$

$$\therefore \tan \tfrac{1}{2} u = \frac{y}{r+x} = \frac{\sqrt{r^2 - x^2}}{r+x} = \sqrt{\frac{r-x}{r+x}} \quad . \quad . \quad . \quad (a)$$

NOTE.—This result can be obtained from [36] by making $A = u$ and $\cos u = \frac{x}{r}$.

Examples. 1. Required the angle of elevation (CAB, Fig. 15) of a railway track at a point where a section of the track (AB) 3967.6 ft. in length subtends a horizontal distance (AC) of 3963.3 ft.

Using (a), we have

$$AB - AC = 4.3, \quad AB + AC = 7930.9.$$

$$\begin{aligned} \log 4.3 &= 0.63347 \\ \text{colog } 7930.9 &= 6.10068 - 10 \\ 2 &\underline{)16.73415 - 20} \\ \log \tan \tfrac{1}{2} CAB &= 8.36708 - 10 \end{aligned}$$

$$\therefore \tfrac{1}{2} CAB = 1^\circ\, 20.035',$$

$$\therefore \quad CAB = 2^\circ\, 40.07'.$$

NOTE.—If we try to find CAB by $\cos CAB = AC \div AB$, we will obtain $\log \cos CAB = 9.99952$, from which we can only determine by the tables that CAB lies between $2^\circ\, 40'$ and $2^\circ\, 43'$.

2. If the lower end of a vibrating pendulum rises and falls through a vertical height of .1 inch, through what angle does the pendulum swing, if it is 50 inches in length?

51. To find the area of a right triangle.

I. *When the two legs are given.*

By Geometry, the area of a triangle is equal to one-half of the product of the base by the altitude.

Hence, denoting the area by K,

$$K = \tfrac{1}{2}xy \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [14]$$

II. *When one leg and the opposite angle are given.*

Since $\frac{x}{y} = \cot u,\ x = y \cot u.$

Substituting in [14], $K = \tfrac{1}{2} y^2 \cot u.$

Similarly, $K = \tfrac{1}{2} x^2 \cot v.$

III. *When one leg and the adjacent angle are given.*

Since $\frac{y}{x} = \tan u,\ y = x \tan u.$

Substituting in [14], $K = \tfrac{1}{2} x^2 \tan u.$

Similarly, $K = \tfrac{1}{2} y^2 \tan v.$

52. Exercise VIII.

1. Prove $K = \tfrac{1}{2} r^2 \sin u \cos u = \tfrac{1}{2} r^2 \sin v \cos v.$

2. Prove $K = \tfrac{1}{2} y \sqrt{(r+y)(r-y)} = \tfrac{1}{2} x \sqrt{(r+x)(r-x)}.$

NOTE. — In computing the value of K by logarithms it will be most convenient to find $2K$ first, and then take half of it.

3. Given $u = 19° 36'$, $y = 11.089$; find $K = 172.7.$

4. Given $u = 40° 30'$, $x = 12.42$; find $K.$

5. Given $x = 14.942$, $y = 7.629$; find $K = 56.996.$

6. Given $r = 12.45$, $x = 8.067$; find $K.$

7. Given $u = 30° 56.3'$, $r = 230.442$; find $K = 11709.$

8. Given $v = 63° 42'$, $x = .0625$; find $K.$

9. Given $K = 15$, $y = 5$; find $r = 7.81$, $u = 39° 48'.$

10. Given $K = 225$, $r = 33$; find $y = 15.42$, $x = 29.18$, $u = 27° 52'.$

11. Given $K = 40$, $u = 21° 41'$; find y, x, and r.

53. Isosceles Triangles. An isosceles triangle is divided into two equal right triangles by drawing a perpendicular from the vertex to the base. Therefore an isosceles triangle is determined by any two parts that determine one of these right triangles.

(1) In the isosceles triangle ABC, given $AB = BC = 14.3$, $AC = 11$; find the angles A, B, C.

Draw $BD \perp AC$, and denote AB by c, BC by a, AC by b, AD by x, and BD by y.

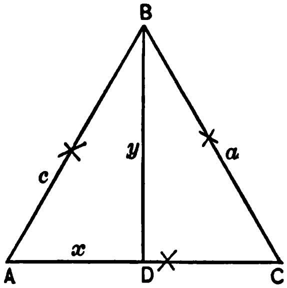

Fig. 16.

(1) $x = \frac{1}{2} b = 5.5.$

(2) $\cos A = \frac{x}{c}$, $\therefore\ A = \cos^{-1}\frac{5.5}{14.3}$

$= 67° 23' = C.$

(3) $A + B + C = 180°$,

$\therefore\ B = 180° - 2A = 45° 14'.$

Also (4) $y = 14.3 \times \sin A = 13.2.$

54. Exercise IX.

The following six examples refer to Fig. 16.

1. Given $a = 18$, $A = 30°$; find $y = 9$, $b = 31.18$, $B = 120°$.

2. Given $a = 7.32$, $y = 7.2$; find $A = 79° 37'$, $B = 20° 46'$, $b = 2.64$.

3. Given $b = 9$, $y = 20$; find $A = 77° 19'$, $B = 25° 22'$, $a = 20.5$.

4. Given $B = 84° 47'$, $c = 91.033$; find $b = 122.75$.

Denoting the area by K, show that

5. $K = \frac{1}{4} b\sqrt{4a^2 - b^2}$. 6. $K = a^2 \sin \frac{1}{2} B \cos \frac{1}{2} B$.

7. Find the side $(2R \sin 22\frac{1}{2}°)$, and apothem $(R \cos 22\frac{1}{2}°)$ of a regular octagon inscribed in a circle whose radius is R.

8. Find the side, apothem, and area of a regular decagon inscribed in a circle whose radius is R.

9. The legs of a pair of dividers are 5 in. long and form an angle of 80° 24.4′. Find the distance (= 6.455 in.) between the points.

10. A wedge measures 18 in. along the side, and its base is 3 in. wide. Find the angle (9° 34′) at its vertex.

11. The pitch of a roof ($\angle BAD$, Fig. 16) is 26° 34′. Show that the height (BD) is about $\frac{1}{4}$ of the width (AC).

MISCELLANEOUS EXERCISES

55. Exercise X.

1. If the base of a right triangle is 63.06 and the acute angle at the base is 72° 52′, what is the perpendicular?

2. Wishing to know the height (*CB*) of a tree standing on a horizontal plane (Fig. 17), I measured from the tree the horizontal line *CA*, 150 ft., and found the angle of elevation, *CAB*, to the top of the tree to be 35° 20′. Required the height.

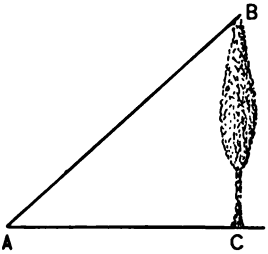

FIG. 17.

3. If the perpendicular of a right triangle is 100 ft., and the angle at the base is 12°, what is the base?

4. If the height of a tree is 80.9 ft., at what horizontal distance from its foot must an observer stand so that the angle of elevation of the top of the tree may be 38°?

5. If the perpendicular of a right triangle is 82 and the base is 95, what is the angle at the base?

6. What is the angle of elevation of the sun when a flag-pole whose height is 138.32 ft. casts a shadow 223.27 ft. in length?

7. From the top (*B*) of a tower 219 ft. high (Fig. 18), the angle of depression, *DBA*, of a stone (*A*) is observed to be 21° 46′. Find the horizontal distance (*CA*) of the stone from the foot of the tower.

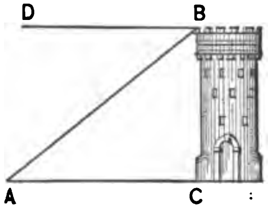

FIG. 18.

8. One of the legs of a right triangle is $\frac{5}{8}$ of the hypotenuse. Find the acute angles of the triangle.

9. One of the acute angles of a right triangle is double the other, and the hypotenuse is 8. Find the legs.

10. The two acute angles of a right triangle are as 4 to 5, and the base is 120. Find the other sides.

11. Find the area of a regular dodecagon inscribed in a circle whose radius is *R*.

12. If the radius of a circle is 30 in., and a chord is 44 in., what angle at the centre is subtended by the chord?

13. I wish to describe an arc of 37° 28′ so as to subtend a chord of 100 ft. Required the radius.

14. Find the area of an isosceles triangle (Fig. 16) in terms of the altitude (y) and the angle at the vertex (B).

15. Find the two acute angles (u, v) of a right triangle, having given the hypotenuse (26) and the perpendicular (12), drawn from the right angle to the hypotenuse.

16. Two towns (A, B) are 8 miles apart, and a man in a balloon directly above A observes the angle of depression of B to be 10° 14′. Find the height of the balloon.

17. A regular heptagon is inscribed in a circle whose radius is 6.6. Find a side and the area of the heptagon.

18. The side of a regular dodecagon is 11.2. Find the radius of the inscribed and circumscribed circles.

19. The base, perpendicular, and hypotenuse of a right triangle are in arithmetical progression; that is, $r - y = y - x$. Find the acute angles.

20. A person wishing to find the width of a river, selects a point (A), Fig. 19, on the bank just opposite a tree (BP) on the other bank, and finds the elevation (BAP) to be 11° 19′; he then recedes 200 ft. (AC) in a direct line from the tree, and finds the elevation (BCP) to be 8° 8′. Find the width (AB).

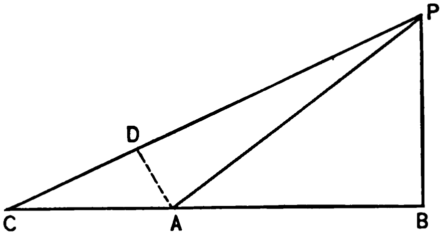

Fig. 19.

Hint. — Draw $AD \perp CP$; find AD, then AP, then AB.

21. A straight horizontal road leads to the foot (B) of a wall 160 ft. high, and from the top (P) of the wall the angles of depression of two men (C, A) on the road are 10° and 8°. Find the distance between the men.

22. Upon the top of a shaft 125 ft. high stands a statue which, at a horizontal distance of 200 ft. from the foot of the shaft, subtends an angle of 3°. How tall is the statue?

23. Two straight roads, AB and AC, cross each other at A, forming an angle of 35°. A surveyor wishes to run a line from a point P in AB perpendicular to AC, so as to get a right triangular lot (PAD) that shall contain $\frac{1}{4}$ of an acre, or 10,890 sq. ft. What will be the frontage of the lot on each road?

24. What would be the frontage on each road, in Ex. 23, if the lot were required to be in the form of an isosceles triangle having its apex at *A*?

25. The horizontal distance between the two extreme positions of the end of a pendulum 36 in. long is 5 in. Through what angle does it swing?

26. The moon is 238,840 mi. from the earth, and subtends an angle of 31.12′. Find its diameter.

27. An engineer in running a railway wishes to pass from the line *AB*, Fig. 20, to the line *DE* on the circular arc *BcD* so that the distance *cC* shall be *f* feet. Required the radius *OB*, the angle of deflection *FCD* being *d* degrees.

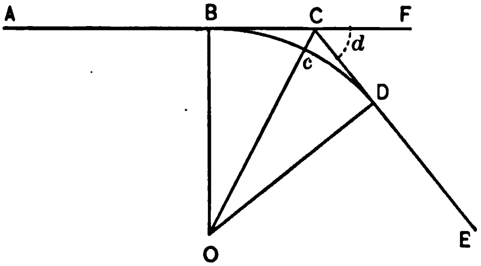

FIG. 20.

28. A chord is stretched around two wheels whose radii are 5 and 17 inches, and whose centres are 24 inches apart. Find the length of the chord.

Answers: **1.** 204.6. **2.** 106.34 ft. **3.** 470.5 ft. **6.** 31° 47′. **7.** 548.46 ft. **9.** 4, 6.928. **11.** $3R^2$. **12.** 94° 20′. **14.** $y^2 \tan \frac{1}{2} B$. **18.** 20.9, 21.64. **19.** 36° 52′, 53° 8′. **21.** 231.05 ft. **22.** 15.07 ft. **23.** 215.3 ft., 176.4 ft. **24.** 194.9 ft. **26.** 2162 mi. **27.** $\frac{f \times \cos \frac{1}{2} d}{1 - \cos \frac{1}{2} d}$. **28.** $24\sqrt{3} + 26\pi$.

CHAPTER III

ANGLES IN GENERAL

DIRECTED LINES AND ANGLES

56. Heretofore we have considered only acute angles and their functions; we now purpose to show that angles may have any magnitude whatever, and that the preceding definitions and formulas are applicable to all angles. In doing so it will be convenient to consider lines and angles in reference to both their magnitudes and directions. When so considered, they are called **Directed Lines and Angles.**

57. Directed Lines. We may suppose the line AB to be generated by a point moving from A to B or from B to A. In the former instance the line is said to be directed from A to B, and is denoted by AB, and in the latter it is directed from B to A, and is denoted by BA; that is, the first letter denotes the initial point and the second the terminal point. Therefore a directed line indicates the distance and direction of the terminal from the initial or starting point.

Fig. 21.

58. Opposite Directions. If a point moves from A to B and then moves backward to C, this opposite motion brings it nearer to A and therefore eliminates a part of the distance AB. Hence distances or lines having opposite directions must have contrary signs.

Fig. 22.

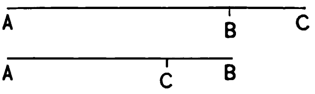

Fig. 23.

Let it be observed that, whether C is on the right or left of B, we have

$$AB + BC = AC \quad . \quad . \quad [15]$$

In the first case AB and BC have the same direction and therefore like signs, and in the second case they have opposite directions and therefore contrary signs. This [15], and all similar equations, may be interpreted thus: *if a point moves from A to B and then from B to C, its distance and direction from A will be AC.*

59. Positive and Negative Lines. Either of two opposite directions may be considered positive, and the other negative; but it is usual to regard distances measured or directed horizontally to the right or vertically up as positive, and therefore those measured horizontally to the left or vertically down as negative.

Thus, let A_3A_1, Fig. 24, be a horizontal, and A_4A_2 a vertical line crossing each other at O, and dividing the surface about O into four quadrants numbered I, II, III, IV, as in the figure. Let P_1P_4 and P_2P_3 be $\perp A_1A_3$. Then BP_1, B_1P_2, and OB are positive, and BP_4, B_1P_3, and OB_1 are negative.

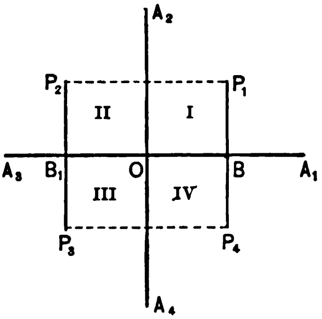

FIG. 24.

60. Addition of Lines. Two or more segments of a straight line are added by placing the initial point of the second upon the terminal point of the first, the initial of the third upon the terminal of the second, and so on; and the sum of all the segments so added, both as to magnitude and direction, is the segment that reaches from the first initial to the last terminal point. When a positive segment is added, the terminal point moves forward; but when a negative segment is added, it moves backward.

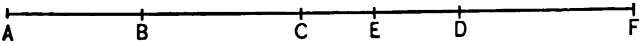

FIG. 25.

Thus,
$$AB + BA = 0. \qquad AB + BC = AC.$$
$$AB + BC + CA = 0. \qquad AB + BC + CE = AE.$$
$$AB + BC + CD + DE + EF + FA = 0.$$

61. COR. I. $AB = -BA$; that is, *changing the order of the letters changes the sign.*

62. Angles of Any Size. The idea of an angle, as usually presented in Geometry, is not sufficiently general for the purposes of Trigonometry. We need here a conception that will admit of positive or negative angles of any magnitude. Such a conception of angles, or angular quantity, may be formed in the following manner:

Let A_1A_3 and A_2A_4, Fig. 26, be perpendicular diameters of a circle whose centre is O and whose radius $OP = r$.

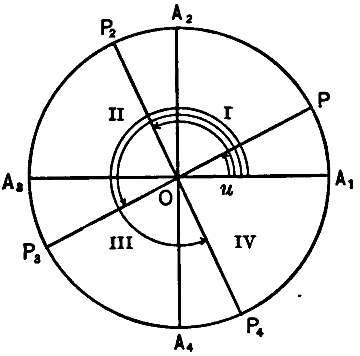

FIG. 26.

Let $u = A_1OP$. Suppose the radius OP to start from the position OA_1 and rotate about the point O as a pivot, like the hand of a clock turning backward.

(1) At the starting-point, $u = A_1OA_1 = 0°$.
(2) When OP reaches OA_2, $u = A_1OA_2 = 90°$.
(3) When OP reaches OP_2, $u = A_1OP_2 = 90° + A_2OP_2$.
(4) When OP reaches OA_3, $u = A_1OA_3 = 180°$.
(5) When OP reaches OP_3, $u = A_1OP_3 = 180° + A_3OP_3$.
(6) When OP reaches OA_4, $u = A_1OA_4 = 270°$.
(7) When OP reaches OP_4, $u = A_1OP_4 = 270° + A_4OP_4$.
(8) When OP reaches OA_1 again, $u = A_1OA_1 = 360°$.
(9) When OP reaches OA_2 again, $u = A_1OA_2 = 450°$.
(10) When OP reaches OP_2 again, $u = A_1OP_2 = 450° + A_2OP_2$.
etc. etc. etc.

Hence, by regarding an angle as the measure of rotation, we have a simple conception that admits of angles of any size we please.

63. The fixed line OA_1, from which the rotation is supposed to commence, is called the **Initial Line**, and the rotating radius in its final position is called the **Terminal Line**.

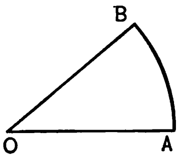

FIG. 27.

64. Directed Angles. The angle AOB may be generated by a line rotating about O from OA to OB or from OB to OA. In the former instance the angle is directed from OA to OB, and is denoted by AOB; in the latter it is directed from OB to OA, and denoted by BOA.

65. Positive and Negative Angles. It is evident from Art. 58 that angles which are generated by opposite rotations have opposite signs. While either of two opposite rotations may be positive, it is usual to regard *counter-clockwise* rotation as *positive* and *clockwise* rotation as *negative*. Thus, Fig. 27, if the size of the $\angle O$ is 30°, then $AOB = +30°$ and $BOA = -30°$.

FUNCTIONS OF ANGLES IN GENERAL

66. Quadrant Angles.

Angles between 0° and 90° are angles of Quadrant I.

Angles between 90° and 180° are angles of Quadrant II.

Angles between 180° and 270° are angles of Quadrant III.

Angles between 270° and 360° are angles of Quadrant IV.

In general, if OA_1, Fig. 26, is the initial side of an angle, as u, the angle, whether positive or negative, will be an angle of Quadrant I, II, III, or IV, according as its terminal side lies between OA_1 and OA_2, OA_2 and OA_3, OA_3 and OA_4, or OA_4 and OA_1.

Thus, the angles

57°, 369°, 745°, $-300°$, are first-quadrant angles.

100°, 500°, $-200°$, $-575°$, are second-quadrant angles.

67. Functions of Quadrant Angles. We will now determine the functions of angles of all the quadrants.

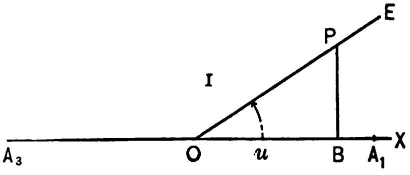

FIG. 28.

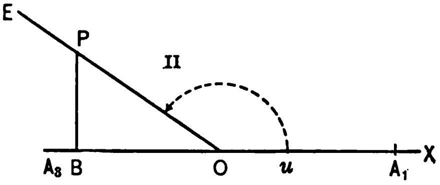

FIG. 29.

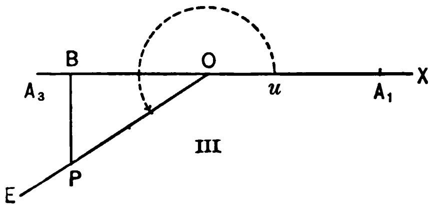

FIG. 30.

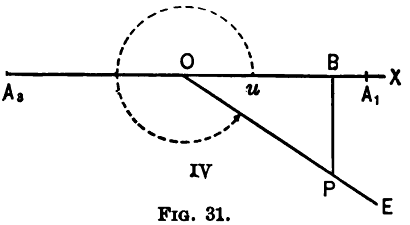

FIG. 31.

In each of the above figures OX and OE are the sides of the $\angle u$, P any point of the side OE, and PB is $\perp OX$. Therefore by Art. 8, we have, in each case,

$$\left.\begin{array}{ll} \sin u = \dfrac{BP}{OP} & \cot u = \dfrac{OB}{BP} \\ \cos u = \dfrac{OB}{OP} & \sec u = \dfrac{OP}{OB} \\ \tan u = \dfrac{BP}{OB} & \csc u = \dfrac{OP}{BP} \end{array}\right\} \qquad (a)$$

68. Signs of the Functions of Quadrant Angles. Regarding $OP \, (= r)$ as always positive, we have

In Fig. 28, or I, $OP = r,\ OB = +x,\ BP = +y;$
In Fig. 29, or II, $OP = r,\ OB = -x,\ BP = +y;$
In Fig. 30, or III, $OP = r,\ OB = -x,\ BP = -y;$
In Fig. 31, or IV, $OP = r,\ OB = +x,\ BP = -y.$

Substituting these values of OP, OB, BP in (a), Art. 67, we find that for all angles:

In **Quadrant I, all the functions** *are* **positive;**
In **Quadrant II, sin** *and* **csc** *are* **positive** *and the others negative;*
In **Quadrant III, tan** *and* **cot** *are* **positive** *and the others negative;*
In **Quadrant IV, sec** *and* **cos** *are* **positive** *and the others negative.*

69. Generality of the Fundamental Formulas. In each quadrant the functions are derived in precisely the same manner from the sides of the right triangle OBP. Hence, formulas [1] to [13] are true of any angle whatever.

70. COR. I. When the **sin** and **cos** of *any* angle are known, we may find (see the Trigonometric Circle):

(1) *The* **tan,** *by dividing the* **sin** *by the* **cos**;
(2) *The* **cot,** *by dividing the* **cos** *by the* **sin**;
(3) *The* **sec,** *by dividing the* **tan** *by the* **sin**;
(4) *The* **csc,** *by dividing the* **cot** *by the* **cos.**

Thus, if $\sin u = \frac{16}{65}$, and $\cos u = -\frac{63}{65}$,

$$\begin{aligned} \tan u &= \frac{16}{65} \div -\frac{63}{65} = -\frac{16}{63}; \\ \cot u &= -\frac{63}{65} \div \frac{16}{65} = -\frac{63}{16}; \\ \sec u &= -\frac{16}{63} \div \frac{16}{65} = -\frac{65}{63}; \\ \csc u &= -\frac{63}{16} \div -\frac{63}{65} = +\frac{65}{16}. \end{aligned}$$

71. Line Values of the Functions. The functions are arithmetical fractions or numbers, but they may be represented relatively by lines (Art. 5) as follows:

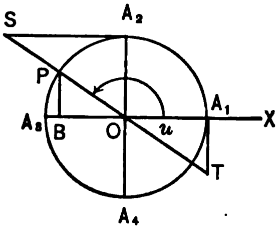

FIG. 32.

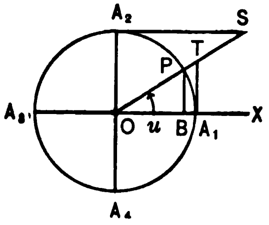

FIG. 33.

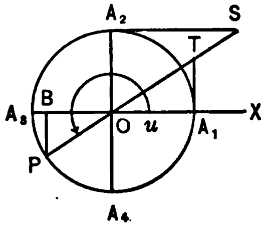

FIG. 34.

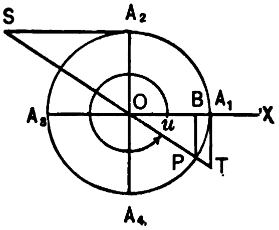

FIG. 35.

In each of the figures, 32 to 35, let the radius of the circle be the unit of length* or 1, $u = A_1OP$, and let BP and A_1T be $\perp$ A_1A_3, and $A_2S \perp A_2A_4$. Then

$$\sin u = \frac{BP}{OP} = \frac{BP}{1} = \text{the line } BP;$$

$$\cos u = \frac{OB}{OP} = \frac{OB}{1} = \text{the line } OB;$$

$$\tan u = \frac{BP}{OB} = \frac{A_1T}{OA_1} = \frac{A_1T}{1} = \text{the line } A_1T;$$

$$\cot u = \frac{OB}{BP} = \frac{A_2S}{OA_2} = \frac{A_2S}{1} = \text{the line } A_2S;$$

$$\sec u = \frac{OP}{OB} = \frac{OT}{OA_1} = \frac{OT}{1} = \text{the line } OT;$$

$$\csc u = \frac{OP}{BP} = \frac{OS}{OA_2} = \frac{OS}{1} = \text{the line } OS.$$

Hence, remembering that only the relative values of lines are to be considered, that of the radius being 1, the functions of the angle formed by OA_1 and a moving radius may be defined thus:

---

* A circle whose radius is the unit of length is called the unit circle.

The **Sine** is the perpendicular to A_1A_3 drawn from the terminal point of the moving radius.

The **Cosine** is the line drawn from the centre of the circle to the foot of the sine.

The **Tangent** is the perpendicular to A_1A_3 drawn from the point A_1 to the moving radius produced.

The **Cotangent** is the perpendicular to A_2A_4 drawn from the point A_2 to the moving radius produced.

The **Secant** and the **Cosecant** are, respectively, the distances from the centre of the circle to the terminal points of the tangent and cotangent.

With regard to the signs of the functions, it follows from Art. 59 (1) that the sin and tan are positive or negative according as they lie above or below A_1A_3; (2) the cos and cot are positive or negative according as they lie on the right or left of A_2A_4; and (3) the sec and csc are positive or negative according as their reciprocals (cos and sin) are, or according as their directions are the same as, or opposite to, the direction of the moving radius.

These are called line values of the functions, as distinguished from the ratio values as given in Art. 67. When properly interpreted the line values are the same as the ratio values, yet each of the former is represented by a *single line*, while each of the latter is the ratio of *two lines*. Hence the former are more easily grasped, and will, therefore, be employed in the articles which immediately follow.

72. How the functions of an angle vary as the angle increases. In Figs. 32 to 35, let us suppose the $\angle u$ to increase continuously from 0° to 360°, and consider the changes in the values of its functions.

NOTE.—Before beginning, let it be observed that the values to be considered are algebraic; hence, when a function is negative it decreases as its absolute value increases, and *vice versa*. Thus, $0 > -1 > -2$, etc., and $-2 < -1 < 0$.

For brevity, + and – will be used to denote *positive* and *negative*.

(1) The **Sine**. The sine PB in Quadrant I is +, and increases from 0 to OA_2 or 1; in II it remains + and decreases from 1 to 0; in III it is –, and decreases from 0 to OA_4 or -1; in IV it is –, and increases from -1 to 0.

COR. I. $\sin 0° = 0, \sin 90° = 1, \sin 180° = 0, \sin 270° = -1$.

(2) The **Cosine**. The cosine OB in Quadrant I is +, and decreases from OA_1 or 1 to 0; in II it is –, and decreases from 0 to OA_3 or -1; in III it is –, and increases from -1 to 0; in IV it is +, and increases from 0 to 1.

Cor. I. $\cos 0° = 1, \cos 90° = 0, \cos 180° = -1, \cos 270° = 0.$

Cor. II. The greatest possible value of sin or cos is 1, and the least is -1, and either of them may have any value between $+1$ and -1.

Cor. III. In one complete revolution of OP the sin and cos each change signs two times.

(3) The **Tangent.** The tangent A_1T in Quadrant I is $+$, and increases from 0 to ∞. As u passes through 90°, the tan jumps from $+\infty$ to $-\infty$; therefore in II it is $-$, and increases from $-\infty$ to 0; in III it is $+$, and increases from 0 to ∞. As u passes through 270°, the tan jumps from $+\infty$ to $-\infty$; therefore in IV it is $-$, and increases from $-\infty$ to 0.

Cor. I. $\tan 0° = 0, \tan 90° = \infty, \tan 180° = 0, \tan 270° = \infty.$

(4) The **Cotangent.** The cotangent A_2S in Quadrant I is $+$, and decreases from ∞ to 0; in II it is $-$, and decreases from 0 to $-\infty$. As u passes through 180°, the cot jumps from $-\infty$ to $+\infty$; therefore in III it is $+$, and decreases from $+\infty$ to 0; in IV it is $-$, and decreases from 0 to $-\infty$.

Cor. I. $\cot 0° = \infty, \cot 90° = 0, \cot 180° = \infty, \cot 270° = 0.$

Cor. II. The greatest possible value of tan or cot is $+\infty$, and the least is $-\infty$, and either of them may have any value between $+\infty$ and $-\infty$; that is, any finite value whatever.

Cor. III. In one complete revolution of OP, the tan and cot each change signs four times.

The following table gives the values of all the functions of 0°, 90°, 180°, 270°, 360°, and how they vary in quadrants I, II, III, and IV. Let I be read, "increases as the angle increases," and D "decreases as the angle increases."

Angle =	0°	Between 0° and 90°	90°	Between 90° and 180°	180°	Between 180° and 270°	270°	Between 270° and 360°	360°
sin....	0	+, I	1	+, D	0	−, D	− 1	−, I	0
cos....	1	+, D	0	−, D	− 1	−, I	0	+, I	1
tan....	0	+, I	$\pm\infty$	−, I	0	+, I	$\pm\infty$	−, I	0
cot....	$\pm\infty$	+, D	0	−, D	$\pm\infty$	+, D	0	−, D	$\pm\infty$
sec....	1	+, I	$\pm\infty$	−, I	− 1	−, D	$\pm\infty$	+, D	1
csc....	$\pm\infty$	+, D	1	+, I	$\pm\infty$	−, I	− 1	−, D	$\pm\infty$

CIRCULAR OR RADIAN MEASURE OF ANGLES

73. The method of estimating angles by degrees is that which is most commonly used in *practical* calculations. We will now explain another method, called the **Circular** or **Radian Measure** of angles, which is of great importance in the *theory* of mathematics.

A **Radian** (rdn.) is an angle whose subtending arc is equal in length to the radius with which the arc is described.

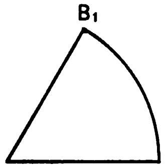

FIG. 36.

74. To find the value of a radian.

Let the circular arc A_1B_1 be equal to its radius OA_1 $(=r)$, then the $\angle A_1OB_1$ is a radian. Since angles at the centre of a circle are proportional to their intercepted arcs, we have

$$\frac{\text{angle } A_1OB_1}{4 \text{ right } \angle s} = \frac{\text{arc } A_1B_1}{\text{circumference of the circle}} = \frac{r}{2\pi r} = \frac{1}{2\pi};$$

$$\therefore \text{ angle } A_1OB_1 = \frac{4 \text{ right } \angle s}{2\pi}, \text{ or a rdn.} = \frac{2 \text{ right } \angle s}{\pi} \quad . \quad . \quad . \quad . \quad (a)$$

where $\pi (= 3.14159 \cdots)$ is the ratio of the circumference of a circle to the diameter.

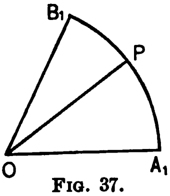

FIG. 37.

75. To find the value of an angle in radian measure.

Let A_1OP be any $\angle$ and A_1OB_1 a radian, and let l and r be the relative lengths of the arcs A_1P and A_1B_1 $(= OA_1)$. Then, by Geometry,

$$\frac{A_1OP}{A_1OB_1} = \frac{\text{arc } A_1P}{\text{arc } A_1B_1}, \text{ or } \frac{A_1OP}{1 \text{ rdn}} = \frac{l}{r}.$$

Now, denoting the relative values of A_1OP and 1 rdn by u and 1, we have

$$u = \frac{l}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [16]$$

If $l = r$, then $u = 1$, which is 1 rdn; if $l = \frac{2}{3} r$, then $u = \frac{2}{3}$, which means (1) that u is $\frac{2}{3}$ of 1 rdn, or that u is an angle whose arc is $\frac{2}{3}$ of the radius; etc., etc.

76. To reduce radian to degree measure, and vice versa.

By (a), Art. 74, we have

(1) $$1 \text{ rdn} = \frac{180°}{\pi}, \text{ or } \angle 1 = 57.29578°, \textit{ nearly} \quad . \quad . \quad [17]$$

(2) $$180° = \pi \text{ rdn, or } 1° = \angle \frac{\pi}{180} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [18]$$

Hence, to reduce degrees to radians, *multiply the number of degrees by* $\frac{\pi}{180}$; and to reduce radians to degrees, *multiply the number of radians by* $\frac{180}{\pi}$ or 57.29578.

Thus, the circular measure of an angle of 90° is $\frac{90 \times \pi}{180}$ or $\frac{\pi}{2}$; of an angle of 60°, $\frac{60 \times \pi}{180}$ or $\frac{\pi}{3}$.

Again, the number of degrees in an angle whose circular measure is $\frac{\pi}{4}$, is $\frac{\pi}{4} \times \frac{180}{\pi} = 45$; in an angle whose circular measure is $\frac{1}{2}$, is $\frac{1}{2} \times 57.29578 = 28.64789$.

77. Exercise XI.

Express each of the following in radian measure:

1. 15°.
2. 25°.
3. 67° 30′.
4. 163° 7′ 30″.
5. 340°.
6. 495°.
7. 53° 20′.
8. 88° 53′ 20″.

Express each of the following in degree measure:

9. $\frac{2\pi}{3}$.
10. $\frac{15\pi}{16}$.
11. $\frac{\pi + 1}{6}$.
12. $\frac{3\pi + 2}{5}$.
13. $\frac{1}{4}$.
14. $\frac{5}{2}$.
15. 1.
16. $\sqrt{2}$.

17. How many seconds in 1 radian?

18. Find the difference in degree measure between 22 radians and 14 right angles.

19. Find the difference in radian measure between 355 radians and 226 right angles.

20. What angle in radian measure does the hour hand of a clock describe in 39 min. $22\frac{1}{2}$ sec.?

21. How long does it take the minute hand of a clock to turn through $1\frac{3}{8}$ radians?

22. A wheel makes 10 revolutions a second. How long does it take to turn through 2 radians, taking $\pi = 3\frac{1}{7}$?

23. Express in radians one of the interior angles of a regular (1) pentagon; (2) octagon.

Answers: **1.** $\frac{\pi}{12}$. **3.** $\frac{3\pi}{8}$. **4.** $\frac{29\pi}{32}$. **6.** $\frac{11\pi}{4}$. **9.** 120°. **11.** 39° 32.96′+. **12.** 130° 55′ 5.9″+. **13.** 14° 19′ 26.2″+. **16.** 81° 1′ 39.69″+. **18.** 30′ 25.7″+. **20.** $\frac{7\pi}{64}$. **22.** $\frac{7}{220}$ sec.

LIMITING VALUES OF $\frac{\sin u}{u}$ AND $\frac{\tan u}{u}$

78. To prove that $\sin u < u < \tan u$ when $u < \frac{\pi}{2}$, u being expressed in circular measure.

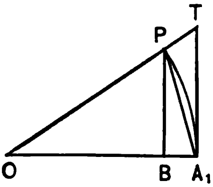

FIG. 38.

Let A_1OP be any acute angle; from O as a centre with a radius $OP(=1)$ describe the arc $A_1P\,(=u)$, and draw PB and $TA_1 \perp OA_1$; then

$$BP = \sin u,\ A_1T = \tan u.$$

The $\triangle\ OA_1P$ is less than the sector OA_1P, being a part of it, and for the same reason the sector OA_1P is less than the $\triangle\ OA_1T$. By Geometry, with respect to area:

$$\triangle\ OA_1P = \tfrac{1}{2}\,OA_1 \times BP,\ \text{sector } OA_1P = \tfrac{1}{2}\,OA_1 \times \text{arc } A_1P,$$

$$\triangle\ OA_1T = \tfrac{1}{2}\,OA_1 \times A_1T.$$

$$\therefore\ \tfrac{1}{2}\,OA_1 \times BP < \tfrac{1}{2}\,OA_1 \times \text{arc } A_1P < \tfrac{1}{2}\,OA_1 \times A_1T \quad . \quad . \quad (a)$$

$(a) \div \tfrac{1}{2}\,OA_1$, $\qquad BP < \text{arc } A_1P < A_1T$,

or $\qquad \sin u < u < \tan u. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$

COR. I. Dividing (b) by $\sin u$,

$$1 < \frac{u}{\sin u} < \sec u,$$

or

$$1 > \frac{\sin u}{u} > \cos u,$$

That is, the value of $\sin u$ is less than u and greater than $u \cos u$.

79. To prove that the limits of $\frac{\sin u}{\tan u}$, $\frac{\tan u}{u}$, $\frac{\sin u}{u}$, when u is indefinitely diminished, are each unity.

In Fig. 38, as the angle A_1OT diminishes the three lines PB, arc PA_1, and TA_1 evidently become more and more nearly equal to one another, and since

$$\frac{PB}{TA_1} = \frac{\sin u}{\tan u} = \cos u,$$

the limit of $PB \div TA_1$ is the same as the limit of $\cos u$, which is ($\cos 0 = 1$) unity.

As the arc A_1P or u is intermediate in value between $PB\,(=\sin u)$ and $TA_1\,(=\tan u)$, evidently

$$\text{limit of } \frac{\tan u}{u} = \text{limit of } \frac{\sin u}{u} = \text{limit of } \frac{\sin u}{\tan u} = 1.$$

REDUCTION OF FUNCTIONS

80. The Mathematical Tables give the functions of positive acute angles only, and it is now to be shown how the functions of any angle may be reduced so that they may be found by the tables.

81. Functions of $-u$ in terms of the functions of u.

Let OA_1 be the radius of a unit circle, and let $u = A_1OP$ and $-u = A_1OP'$. Whatever may be the size of u, the two points P and P' will be (1) on the same side of A_2A_4 and at equal distances from it, and (2) on opposite sides of A_1A_3 and at equal distances from it. Hence, u and $-u$ have the same cosine, OB, and numerically equal sines, BP and BP', with opposite directions.

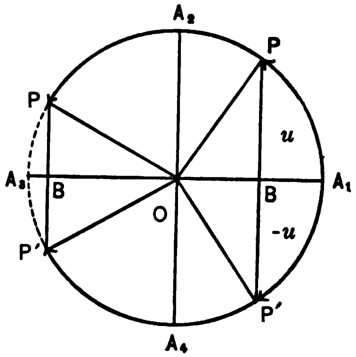

FIG. 39.

$$\therefore \sin -u = -\sin u, \text{ and } \cos -u = \cos u \quad . \quad . \quad . \quad [19]$$

Whence, Art. 70,

$$\tan -u = -\tan u, \ \cot -u = -\cot u,$$

$$\sec -u = \sec u, \ \csc -u = -\csc u.$$

Since these formulas are true for any value of u, they enable us to express the functions of any negative angle in terms of the functions of an equal positive angle.

82. Functions of $90° + u$ in terms of the functions of u.

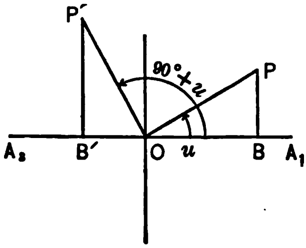

FIG. 40.

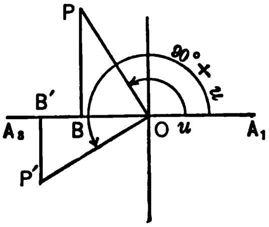

FIG. 41.

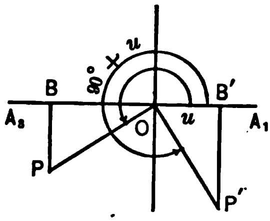

FIG. 42.

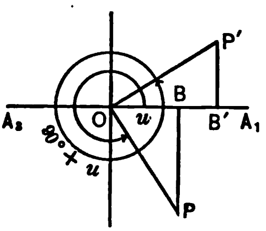

FIG. 43.

In each of these four figures let $OP = OP' = 1$, and let $u = \angle A_1OP'$ be an angle of Quadrant I, II, III, or IV. Let OP' be $\perp$ OP, and draw PB and $P'B' \perp A_1A_3$. Then in each figure,

$$\sin u = BP, \ \sin(90 + u) = B'P',$$

and $$\cos u = OB, \ \cos(90 + u) = OB'.$$

The two $\triangle$ OBP and $OB'P'$ are evidently equal.

$$\therefore \ P'B' = OB, \text{ or } \mathbf{sin}\,(\mathbf{90°} + u) = +\,\mathbf{cos}\,u \quad . \quad . \quad [20]$$

$$OB' = -PB,^{*} \text{ or } \mathbf{cos}\,(\mathbf{90°} + u) = -\,\mathbf{sin}\,u \quad . \quad . \quad [21]$$

Whence, Art. 70, $\tan(90° + u) = -\cot u,$

$$\cot(90° + u) = -\tan u,$$

$$\sec(90° + u) = -\csc u,$$

$$\csc(90° + u) = +\sec u.$$

Formulas [20] and [21] may be stated thus: *the sine and cosine of any angle are equal to, respectively, the cosine and minus the sine of an angle* 90° *less.*

Thus, $\sin 290° = \cos 200° = -\sin 110° = -\cos 20°;$

and $\cos 155° = -\sin 65° = -\cos -25° = -\cos 25°.$

In this manner the sine and cosine of any angle may be reduced to the sine or cosine of a positive angle not greater than 45°; but by the next article, the reductions may be more conveniently effected.

[illegible] Functions of $n \times 90° \pm u$ in terms of the functions of u.

Since sin changes to cos and cos to sin when 90° is added to, or subtracted from, the angle, it is evident that (omitting the signs)

$$\sin(n \times 90° \pm u) = \sin \pm u \ \text{ or } \ \cos \pm u,$$

and $$\cos(n \times 90° \pm u) = \cos \pm u \ \text{ or } \ \sin \pm u,$$

according as n is an even or odd integer.

---

* Numerically, $OB' = PB$, but they have contrary signs (Art. 59).

Whence, Art. 70,

$$\tan(n \times 90^\circ \pm u) = \tan \pm u \text{ or } \cot \pm u,$$

$$\cot(n \times 90^\circ \pm u) = \cot \pm u \text{ or } \tan \pm u,$$

$$\sec(n \times 90^\circ \pm u) = \sec \pm u \text{ or } \csc \pm u,$$

$$\csc(n \times 90^\circ \pm u) = \csc \pm u \text{ or } \sec \pm u,$$

according as n is an even or odd integer.

Therefore to find any function of any angle, *divide the number of degrees in the angle by 90, and take the same or co-function of the remainder according as the quotient is an even or odd integer, and prefix the sign + or − according to the quadrant in which the given angle lies.*

Thus, to reduce sin 600°, we divide 600 by 90 and find 6 quotient and 60 remainder. Hence, (1) 600° lies in Quadrant III, and therefore its sin is −; and (2), since 6 is an even number, we take the same function of 60°. Thus, $\sin 600^\circ = -\sin 60^\circ$.

84. Exercise XII.

Reduce the following to functions of positive acute angles:

1. $\sin 172^\circ$.
2. $\cos 200^\circ$.
3. $\tan 164^\circ 33'$.
4. $\cot 91^\circ$.
5. $\sec 300^\circ$.
6. $\sin 274^\circ 15'$.
7. $\cos 713^\circ$.
8. $\tan 813^\circ$.
9. $\csc 872^\circ 47'$.
10. $\sin -300^\circ$.
11. $\cot -510^\circ$.
12. $\tan -700^\circ 41'$.

Answers: 1. $\sin 8^\circ$. 3. $-\cot 74^\circ 33'$. 5. $\csc 30^\circ$. 7. $\sin 83^\circ$. 9. $\sec 62^\circ 47'$. 11. $\tan 60^\circ$.

85. Functions of 180° − u in terms of the functions of u.

If $u > 0$ and $u < 180^\circ$, it follows from what has preceded that

$$\sin(180^\circ - u) = \sin u, \quad \cot(180^\circ - u) = -\cot u,$$

$$\cos(180^\circ - u) = -\cos u, \quad \sec(180^\circ - u) = -\sec u,$$

$$\tan(180^\circ - u) = -\tan u, \quad \csc(180^\circ - u) = \csc u.$$

MISCELLANEOUS EXERCISES

86. Exercise XIII.

Give the quadrant in which each of the following angles lies:

1. 143°.
2. 290°.
3. $310^\circ 20'$.
4. 400°.
5. -8°.
6. $1\frac{2}{3}\pi$.
7. -700°.
8. $-2\frac{5}{6}\pi$.

Give the signs of the functions of the following angles:

9. $100°$. **11.** $432° 31'$. **13.** $\frac{7\pi}{4}$. **15.** $\frac{5\pi}{6}$.
10. $300° 10'$. **12.** $-146° 40'$. **14.** $\frac{11\pi}{5}$. **16.** $-1\frac{1}{8}$.

Reduce the following to functions of positive acute angles:

17. $\sin 157°$. **20.** $\csc 804° 35'$. **23.** $\cos -1000°$. **26.** $\tan 600°$.
18. $\cos 322°$. **21.** $\cot -330°$. **24.** $\tan 401° 20'$. **27.** $\csc -320°$.
19. $\sec 559°$. **22.** $\sin -134°$. **25.** $\sec 225°$. **28.** $\cot 512° 42'$.

Prove the following (see Arts. 28, 29):

29. $\tan 135° = -1$. **31.** $\sec 315° = +\sqrt{2}$. **33.** $\sin 300° = -\frac{1}{2}\sqrt{3}$.
30. $\cos 120° = -\frac{1}{2}$. **32.** $\cot 210° = +\sqrt{3}$. **34.** $\csc -240° = +\frac{2}{3}\sqrt{3}$.

What other angle less than 180° has the same sine:

35. As 70°? **36.** As 48° 21′? **37.** As 129° 32′ 12″?

Are there two angles less than 180° that have:

38. The same tan? **40.** The same csc? **42.** The same cot?
39. The same sec? **41.** The same cos?

In what quadrant does the angle u lie:

43. If $\sin u$ is $+$ and $\tan u$ is $-$? **45.** If $\cos u$ is $-$ and $\tan u$ is $+$?
44. If $\cos u$ is $+$ and $\cot u$ is $-$? **46.** If $\sec u$ is $-$ and $\csc u$ is $+$?

Give the number of angles less than 360° and also their quadrants:

47. Whose $\cos = +\frac{1}{2}$. **50.** Whose $\cos = -\frac{1}{2}$. **53.** Whose $\tan = +\frac{1}{3}$.
48. Whose $\sin = -\frac{2}{5}$. **51.** Whose $\sin = +\frac{3}{4}$. **54.** Whose $\sec = -4$.
49. Whose $\csc = +5$. **52.** Whose $\cot = -\frac{1}{9}$. **55.** Whose $\csc = -7$.

Reduce the following and find their values, using the table of natural functions:

56. $\sin 320°$. **59.** $\tan 413° 20'$. **62.** $\cot -240°$. **65.** $\tan \frac{5\pi}{8}$.
57. $\tan 500°$. **60.** $\cot 171°$. **63.** $\sin 640° 50'$. **66.** $\cot \frac{1}{2}$.
58. $\sin -700°$. **61.** $\cos 900°$. **64.** $\sin \frac{3\pi}{4}$. **67.** $\cos 1$.

Find the two least positive values in degree measure of each of the following angles, using the table of natural functions:

68. $\sin^{-1}.09005$. **71.** $\sin^{-1}.98700$. **74.** $\cos^{-1}.28402$.
69. $\cos^{-1}-.97304$. **72.** $\cot^{-1}2.3559$. **75.** $\cot^{-1}-.54673$.
70. $\tan^{-1}.70455$. **73.** $\sin^{-1}-.65935$. **76.** $\tan^{-1}-.97133$.

77. In finding $\sec u$ from $\sec u = \pm\sqrt{1+\tan^2 u}$, when must we use the + sign, and when the − sign?

78. Given $\cos u = -\frac{12}{13}$; find the other functions of u when u is an angle in Quadrant III.

79. Given $\tan u = -2$ and $\cos u$ positive; find the other functions of u.

80. Which functions of any angle of any triangle are always positive? Which functions may be negative, and in what cases?

Prove the following:

81. $\sin 0° + \cos 90° = 0$. **86.** $\cot 45° + \csc 270° = 0$.
82. $\sin 180° + \cos 270° = 0$. **87.** $2\cos 60° + \sec 180° = 0$.
83. $\cos 0° + \sin 90° = 2$. **88.** $2\sin 30° + \tan 60° = 1 + \sqrt{3}$.
84. $\cos 180° + \sin 270° = -2$. **89.** $3\tan 210° + 2\tan 120° = -\sqrt{3}$.
85. $\tan 45° + \cot 90° = 1$. **90.** $5\sec^2 135° - 6\cot^2 300° = 8$.

Find the values of the following:

91. $\frac{\sec u + \sin u}{\csc u + \cos u}$, when $\cot u = -\frac{1}{2}$, and u is in Quadrant II.

92. $\frac{\cos u \cot u - \sin u \tan u}{\csc u - \sec u}$, when $\sin u = -\frac{3}{5}$, and u is in Quadrant III.

Answers: **1.** II. **3.** IV. **6.** IV. **8.** III. **10.** cos and sec positive, the others negative. **12.** All negative except tan and cot. **14.** All positive. **16.** All negative except cos and sec. **17.** sin 23° or cos 67°. **19.** − sec 19°. **21.** tan 60°. **24.** tan 41° 20′. **27.** sec 50°. **35.** 110°. **37.** 50° 27′ 48″. **38.** *No.* **40.** *Yes.* **43.** II. **45.** III. **47.** *Two*, I, IV. **49.** *Two*, I, II. **52.** *Two*, II, IV. **56.** − .64279. **58.** .34202. **63.** − .98218. **65.** − 2.4142. **67.** .54031. **68.** 5° 10′, 174° 50′. **69.** 164° 40′, 193° 20′. **70.** 35° 10′, 215° 10′. **75.** 118° 40′, 298° 40′. **77.** Use + when u is in I or IV, and −, when u is in II or III. **79.** $\cot u = -\frac{1}{2}$, $\sin u = -\frac{2\sqrt{5}}{5}$, $\csc u = -\frac{\sqrt{5}}{2}$, $\cos u = \frac{\sqrt{5}}{5}$, $\sec u = \sqrt{5}$. **91.** −2. **92.** $\frac{37}{25}$.

FUNCTIONS OF SUMS AND SUMS OF FUNCTIONS

87. Functions of $u + v$ in terms of the functions of u and v.

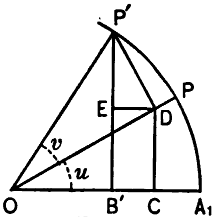

FIG. 44.

Let $A_1OP (= u)$, $POP' (= v)$,

and $A_1OP' (= u + v)$

be acute angles. Draw $P'D \perp$ the radius $OP (=1)$, and DC, $P'B' \perp OA_1$, and $ED \parallel OA_1$, whence $\angle EP'D = u$.

The radius being 1, $P'D = \sin v$, and $OD = \cos v$; and

$$\frac{CD}{OD} = \sin u,\ \frac{OC}{OD} = \cos u,\ \frac{ED}{P'D} = \sin u,\ \frac{EP'}{P'D} = \cos u.$$

$$(1)\quad \sin(u+v) = P'B' = CD + EP'$$

$$= \frac{CD}{OD} \times OD + \frac{EP'}{P'D} \times P'D.$$

$$\therefore\ \sin(u+v) = \sin u \cos v + \cos u \sin v \quad . \ . \ . \quad [22]$$

$$(2)\quad \cos(u+v) = OB' = OC - ED$$

$$= \frac{OC}{OD} \times OD - \frac{ED}{P'D} \times P'D.$$

$$\therefore\ \cos(u+v) = \cos u \cos v - \sin u \sin v \quad . \ . \ . \quad [23]$$

Use. These formulas are used in finding the sine and cosine of the sum of two angles when the sines and cosines of the angles are known.

Thus, if $\sin u = \frac{3}{5}$, $\cos u = \frac{4}{5}$, $\sin v = \frac{5}{13}$, $\cos v = \frac{12}{13}$, then

by [22], $\sin(u+v) = \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} = \frac{56}{65}$;

that is, $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{56}{65}$;

by [23], $\cos(u+v) = \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} = \frac{33}{65}$;

that is, $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$.

88. Generality of [22] and [23]. In deducing formulas [22] and [23], the angles u, v, and $u + v$ were taken, for convenience, less than 90°; we now purpose to show that these important formulas are true for all values of the angles u and v.

I. When u and v are acute, and $u + v > 90°$.

Evidently $90° - u$ and $90° - v$, and their sum, $180° - (u + v)$, are all acute angles.

Hence [22] and [23] are true for these angles. That is,

$$\sin[180° - (u + v)] = \sin(90° - u)\cos(90° - v) + \cos(90° - u)\sin(90° - v),$$

$$\cos[180° - (u + v)] = \cos(90° - u)\cos(90° - v) + (\sin 90° - u)\sin(90° - v).$$

Reducing these by Arts. 82, 85,

$$\sin(u + v) = \sin u \cos v + \cos u \sin v,$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v.$$

Therefore [22] and [23] are true when u and v are any two acute angles.

II. *When u and v are any positive angles.*

Let u and v be any angles for which [22] and [23] are true.

Let $90° + u = u'$, then $90° + u + v = u' + v$.

Taking (1) the sines and then (2) the cosines of both members of these equations, whatever may be the value of u (Art. 82), we have

$$(1) \qquad \cos u = \sin u', \qquad \cos(u + v) = \sin(u' + v),$$

$$(2) \qquad -\sin u = \cos u', \qquad -\sin(u + v) = \cos(u' + v).$$

Substituting these values for $\cos u$, $\cos(u + v)$, $\sin u$, $\sin(u + v)$ in [23] and [22], we have, respectively,

$$\sin(u' + v) = \sin u' \cos v + \cos u' \sin v,$$

$$\cos(u' + v) = \cos u' \cos v - \sin u' \sin v.$$

Therefore, formulas [22] and [23] are true when u or v is increased by 90°, and also true for each repeated increase of one or the other angle by 90°. Hence, since they have been proved true for any two acute angles, they are true for any two positive angles whatever.

III. *When u and v are any negative angles.*

$$\sin(-u - v) = \sin -(u + v) = -\sin(u + v) \qquad \text{by [19]}$$

by [22], $\qquad = -\sin u \cos v - \cos u \sin v$

by [19], $\qquad = \sin -u \cos -v + \cos -u \sin -v.$

In a similar manner it may be shown that [23] is true when u and v are negative.

The case in which the two angles u and v have contrary signs will be considered in Art. 89.

The other functions of $u+v$ may be readily derived from [22] and [23].

Dividing [22] and [23] by $\cos u \cos v$, we have, (*f*) Art. 21,

$$(1)\quad \frac{\sin(u+v)}{\cos u \cos v}=\frac{\sin u \cos v}{\cos u \cos v}+\frac{\cos u \sin v}{\cos u \cos v}=\tan u+\tan v \quad . \quad . \quad (a)$$

$$(2)\quad \frac{\cos(u+v)}{\cos u \cos v}=\frac{\cos u \cos v}{\cos u \cos v}-\frac{\sin u \sin v}{\cos u \cos v}=1-\tan u \tan v \quad . \quad (b)$$

Dividing (a) by (b), remembering that $\frac{\sin(u+v)}{\cos(u+v)}=\tan(u+v)$,

$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u \tan v} \quad . \quad . \quad . \quad . \quad . \quad [24]$$

Dividing [22] and [23] by $\sin u \sin v$, and proceeding as above, remembering that $\cos \div \sin = \cot$, we have

$$\cot(u+v)=\frac{\cot u \cot v-1}{\cot u+\cot v} \quad . \quad . \quad . \quad . \quad . \quad [25]$$

Example. If $\tan u=\frac{1}{2}$ and $\tan v=\frac{1}{3}$, then by [24]

$$\tan(u+v)=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}=\frac{\frac{5}{6}}{1-\frac{1}{6}}=1;$$

that is, $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\tan^{-1}1=45°=\frac{1}{4}\pi.$

89. Functions of $u-v$ in terms of the functions of u and v.

Let v be any angle less than 360°, then $360°-v$ is a positive angle, and we may therefore substitute it for v in [22], which gives

$$\sin(360°+u-v)=\sin u \cos(360°-v)+\cos u \sin(360°-v).$$

Now the angles $360°+u-v$ and $u-v$ have the same terminal, and so have $360°-v$ and $-v$; therefore, $\sin(360°+u-v)=\sin(u-v)$ and $\sin(360°-v)=\sin-v=-\sin v$, $\cos(360°-v)=\cos-v=\cos v$.

$$\therefore \sin(u-v)=\sin u \cos v-\cos u \sin v \quad . \quad . \quad . \quad [26]$$

This formula may be written (Art. 81)

$$\sin(u-v)=\sin u \cos -v+\cos u \sin -v,$$

which shows that [26] may be derived from [22] by substituting $-v$ for v, and this proves the *generality* of [22] and also of [23], since [23] and [22] are interdependent.

Therefore, we may deduce the values of $\cos(u-v)$, $\tan(u-v)$, $\cot(u-v)$ from [23], [24], [25] by substituting $-v$ for v, which gives (Art. 81)

$$\cos(u-v) = \cos u \cos v + \sin u \sin v \quad . \quad . \quad . \quad . \quad [27]$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v} \quad . \quad . \quad [28], \quad \cot(u-v) = \frac{\cot u \cot v + 1}{\cot v - \cot u} \quad . \quad . \quad [29]$$

90. Functions of $2u$ in terms of the functions of u.

Making $v = u$ in [22], we have

$$\sin(u+u) = \sin u \cos u + \cos u \sin u,$$

or

$$\sin 2u = 2 \sin u \cos u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [30]$$

Making $v = u$ in [23], we have

$$\cos(u+u) = \cos u \cos u - \sin u \sin u,$$

or

$$\cos 2u = \cos^2 u - \sin^2 u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [31]$$

Making $v = u$ in [24] and [25], we have

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} \quad . \quad . \quad [32], \qquad \cot 2u = \frac{\cot^2 u - 1}{2 \cot u} \quad . \quad . \quad [33]$$

By these formulas, [30] to [33], the functions of twice an angle are found when the functions of the angle are known.

91. Functions of half an angle in terms of the functions of the angle. Take the formulas

(1) $\cos^2 u + \sin^2 u = 1$ . . . . (*a*)

(2) $\cos^2 u - \sin^2 u = \cos 2u$ . . . . (*b*)

(*a*) + (*b*), $2 \cos^2 u = 1 + \cos 2u$.

(*a*) − (*b*), $2 \sin^2 u = 1 - \cos 2u$.

Putting A in place of $2u$, and therefore $\frac{1}{2}A$ in place of u, we have

$$2 \cos^2 \tfrac{1}{2}A = 1 + \cos A \quad . \quad . \quad . \quad (c), \text{ and } \quad 2 \sin^2 \tfrac{1}{2}A = 1 - \cos A \quad . \quad . \quad . \quad (d)$$

$$\therefore \cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}} \quad . \quad . \quad [34], \quad \sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}} \quad . \quad . \quad [35]$$

By division, Art. 70, we have

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} \quad . \quad . \quad [36], \quad \cot \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{1 - \cos A}} \quad . \quad . \quad [37]$$

The sign before the radical, in each case, is to be determined by the quadrant in which the angle $\frac{1}{2}A$ lies.

Again, putting $\frac{1}{2}A$ for u in [30],

$$2\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A = \sin A \quad . \quad . \quad . \quad . \quad . \quad . \quad [38]$$

Dividing (d), Art. 91, by [38],

$$\tan\tfrac{1}{2}A = \frac{1-\cos A}{\sin A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [39]$$

Dividing (c), Art. 91, by [38],

$$\cot\tfrac{1}{2}A = \frac{1+\cos A}{\sin A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [40]$$

Examples. By Art. 29, $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{1}{2}\sqrt{3}$,

$\therefore$ By [35], $\sin 15° = \sqrt{\dfrac{1-\frac{1}{2}\sqrt{3}}{2}} = \frac{1}{2}\sqrt{2-\sqrt{3}}.$

By [34], $\cos 15° = \sqrt{\dfrac{1+\frac{1}{2}\sqrt{3}}{2}} = \frac{1}{2}\sqrt{2+\sqrt{3}}.$

By [39], $\tan 15° = \dfrac{1-\frac{1}{2}\sqrt{3}}{\frac{1}{2}} = 2-\sqrt{3}.$

92. Sum and difference of functions in terms of the product of functions.

$[22]+[26]$, $\sin(u+v)+\sin(u-v) = 2\sin u\cos v$ . . . (a)

$[22]-[26]$, $\sin(u+v)-\sin(u-v) = 2\cos u\sin v$ . . . (b)

$[23]+[27]$, $\cos(u+v)+\cos(u-v) = 2\cos u\cos v$ . . . (c)

$[23]-[27]$, $\cos(u+v)-\cos(u-v) = -2\sin u\sin v$ . . (d)

Making $u+v=A$ and $u-v=B$, and therefore $u=\frac{1}{2}(A+B)$, and $v=\frac{1}{2}(A-B)$,

From (a), $\sin A+\sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$ . . [41]

From (b), $\sin A-\sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$ . . [42]

From (c), $\cos A+\cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$ . . [43]

From (d), $\cos A-\cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$ . . [44]

93. Useful formulas.

$[41]\div[42]$, $\dfrac{\sin A+\sin B}{\sin A-\sin B} = \dfrac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}$ . . . . . . [45]

$[41]\div[43]$, $\dfrac{\sin A+\sin B}{\cos A+\cos B} = \tan\frac{1}{2}(A+B)$ . . . . . . [46]

$[42]\div[43]$, $\dfrac{\sin A-\sin B}{\cos A+\cos B} = \tan\frac{1}{2}(A-B)$ . . . . . . [47]

In [30] make $u = \frac{1}{2}(A + B)$,

$$\sin(A + B) = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A + B) \quad . \quad . \qquad (a)$$

[41] ÷ (a), $$\frac{\sin A + \sin B}{\sin(A + B)} = \frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} \quad . \quad . \quad . \quad . \quad . \quad . \qquad [48]$$

[42] ÷ (a), $$\frac{\sin A - \sin B}{\sin(A + B)} = \frac{\sin \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)} \quad . \quad . \quad . \quad . \quad . \quad . \qquad [49]$$

In [44] make $A = b - c$, and $B = a$,

$$\cos(b - c) - \cos a = 2 \sin \tfrac{1}{2}(a + b - c) \sin \tfrac{1}{2}(a - b + c) \qquad [50]$$

MISCELLANEOUS EXERCISES

94. Exercise XIV.

Prove the following:

1. $\sin(90° + v) = \cos v$.

Writing 90° for u in [22],

$$\sin(90° + v) = \sin 90° \cos v + \cos 90° \sin v$$
$$= 1 \times \cos v + 0 \times \sin v = \cos v.$$

2. $\cos(90° + v) = -\sin v$.
3. $\cos(180° - v) = -\cos v$.
4. $\cot(90° + v) = -\tan v$.
5. $\tan(180 + v) = \tan v$.
6. $\sin(180° - v) = \sin v$.
7. $\cos(u - 270°) = -\sin u$.
8. $\tan(45° + v) = \dfrac{1 + \tan v}{1 - \tan v}$.
9. $\cot(v - 45°) = \dfrac{1 + \cot v}{1 - \cot v}$.
10. $\sin(60° + v) = \frac{1}{2}(\sqrt{3} \cos v + \sin v)$.
11. $\cos(60 - v) = \frac{1}{2}(\cos v + \sqrt{3} \sin v)$.
12. $\tan(u - 60) = \dfrac{\tan u - \sqrt{3}}{1 + \sqrt{3} \tan u}$.
13. $\cot(u + 30) = \dfrac{\sqrt{3} \cot u - 1}{\cot u + \sqrt{3}}$.
14. $\sin(30° + u) + \sin(30° - u) = \cos u$.
15. $\cos(60° + u) + \cos(60° - u) = \cos u$.
16. $\tan(45° + u) - \tan(45° - u) = 2 \tan 2u$.
17. $\cos 5u \cos 2u + \sin 5u \sin 2u = \cos 3u$.

In [27] put $5u$ for u and $2u$ for v.

18. $\sin(u+v)\sin(u-v)=(\sin u+\sin v)(\sin u-\sin v)$.

[22] × [26], $\sin(u+v)\sin(u-v)=\sin^2 u\cos^2 v-\cos^2 u\sin^2 v$.

Put $1-\sin^2 v$ for $\cos^2 v$,
and $1-\sin^2 u$ for $\cos^2 u$, $=\sin^2 u-\sin^2 v$
$=(\sin u+\sin v)(\sin u-\sin v)$.

19. $\cos(u+v)\cos(u-v)=(\cos u+\sin v)(\cos u-\sin v)$.

20. $\dfrac{\sin 7u+\sin 3u}{\cos 7u+\cos 3u}=\tan 5u$. In [46] make $A=7u$, $B=3u$.

21. $\dfrac{\sin 13^\circ+\sin 7^\circ}{\sin 20^\circ}=\dfrac{\cos 3^\circ}{\cos 10^\circ}$. **22.** $\dfrac{\sin 9^\circ-\sin 5^\circ}{\sin 14^\circ}=\dfrac{\sin 2^\circ}{\sin 7^\circ}$.

23. $\dfrac{\sin 17u+\sin 11u}{\cos 17u-\cos 11u}=-\cot 3u$. Use [41] and [44].

24. $\dfrac{\tan(A+B)-\tan A}{1+\tan(A+B)\tan A}=\tan B$.

25. $\cos^4 u-\sin^4 u=\cos 2u$. **26.** $\tan\frac{1}{2}u=\csc u-\cot u$.

27. $\sin 15^\circ=\frac{1}{4}(\sqrt{6}-\sqrt{2})$. In [26], make $u=45^\circ$, $v=30^\circ$.

28. $\cos 15^\circ=\frac{1}{4}(\sqrt{6}+\sqrt{2})$. In [27], make $u=45^\circ$, $v=30^\circ$.

29. $\cot 15^\circ=2+\sqrt{3}$. In [40], make $A=30^\circ$.

30. $\sin 22\frac{1}{2}^\circ=\frac{1}{2}\sqrt{2-\sqrt{2}}$. In [35], make $A=45^\circ$.

31. $\cos 22\frac{1}{2}^\circ=\frac{1}{2}\sqrt{2+\sqrt{2}}$. In [34], make $A=45^\circ$.

32. $\tan 22\frac{1}{2}^\circ=\sqrt{2}-1$. **33.** $\cot 22\frac{1}{2}^\circ=\sqrt{2}+1$.

34. $\sin 3u=3\sin u-4\sin^3 u$.

$$\begin{aligned}\sin 3u=\sin(2u+u)&=[\sin 2u]\cos u+\sin u[\cos 2u]\\&=[2\sin u\cos u]\cos u+\sin u[\cos^2 u-\sin^2 u]\\&=2\sin u\cos^2 u+\sin u\cos^2 u-\sin^3 u\\&=2\sin u(1-\sin^2 u)+\sin u(1-\sin^2 u)-\sin^3 u\\&=3\sin u-4\sin^3 u.\end{aligned}$$

35. $\cos 3u=4\cos^3 u-3\cos u$.

36. $(\sin u-\sin v)^2+(\cos u-\cos v)^2=(2\sin\frac{1}{2}(u-v))^2$.

37. $\tan^{-1}a+\tan^{-1}b=\tan^{-1}\dfrac{a+b}{1-ab}$.

From [24], we have $u+v=\tan^{-1}\dfrac{\tan u+\tan v}{1-\tan u\tan v}$ . . . . (a)

Let $u=\tan^{-1}a$, $v=\tan^{-1}b$, then $\tan u=a$, $\tan v=b$. Substituting in (a), and we get the given relation.

38. $2\tan^{-1}a = \tan^{-1}\frac{2a}{1-a^2}$. Make $b = a$ in Ex. 37.

39. $\sin 2u = \frac{2\tan u}{1+\tan^2 u}$. 40. $\cos 2u = \frac{1-\tan^2 u}{1+\tan^2 u}$.

Note. — The following six formulas serve to change sums to products, and thus adapt them to logarithmic computation.

Prove the following:

41. $\cot u + \tan u = \frac{2}{\sin 2u}$.

$$\cot u + \tan u = \frac{\cos u}{\sin u} + \frac{\sin u}{\cos u} = \frac{\cos^2 u + \sin^2 u}{\sin u \cos u} = \frac{1}{\sin u \cos u} = \frac{2}{\sin 2u}.$$

42. $\cot u - \tan u = 2\cot 2u$.

43. $\cot u + \tan v = \frac{\cos(u-v)}{\sin u \cos v}$.

$$\cot u + \tan v = \frac{\cos u}{\sin u} + \frac{\sin v}{\cos v} = \frac{\cos u \cos v + \sin u \sin v}{\sin u \cos v} = \frac{\cos(u-v)}{\sin u \cos v}.$$

44. $\cot u - \tan v = \frac{\cos(u+v)}{\sin u \cos v}$.

45. $1 + \tan u \tan v = \frac{\cos(u-v)}{\cos u \cos v}$.

46. $\cot u \cot v - 1 = \frac{\cos(u+v)}{\sin u \sin v}$.

47. Show that $\cos 2u \cos 2v = 2\sin^2 u \sin^2 v + 2\cos^2 u \cos^2 v - 1$.

48. Show that $\tan 3u = \frac{3\tan u - \tan^3 u}{1 - 3\tan^2 u}$.

49. Show that $\cos 4u = 1 - 8\sin^2 u + 8\sin^4 u$.

[Use $4u = 2u + 2u$ in [23].]

50. Show that $\sin 4u = 4\sin u \cos u - 8\sin^3 u \cos u$.

51. Show that $\cos 5u = 5\cos u - 20\cos^3 u + 16\cos^5 u$.

CHAPTER IV

OBLIQUE TRIANGLES

RELATIONS OF SIDES AND ANGLES

95. Notation.—The angles will be denoted by A, B, C, and the opposite sides, respectively, by a, b, c.

96. Relation of one angle to the three sides.

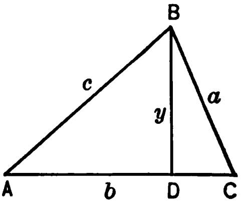

FIG. 45.

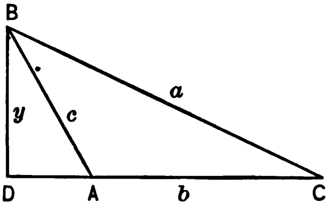

FIG. 46.

Let the angle, say A, be acute (Fig. 45), or obtuse (Fig. 46). In each figure draw $BD \perp AC$, and let $y = BD$. Then in both figures [15],

$$AD + DC = AC, \text{ or } DC = b - AD.$$

$$\therefore\ \overline{DC}^2 = b^2 + \overline{AD}^2 - 2b \times AD.$$

Adding y^2, $\quad \overline{DC}^2 + y^2 = b^2 + \overline{AD}^2 + y^2 - 2b \times AD.$

But $\overline{DC}^2 + y^2 = a^2$, $\overline{AD}^2 + y^2 = c^2$, and since $AD \div AB = \cos CAB$, $AD = c\cos A$.

$$\therefore\ \boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A} \quad \ldots\ldots \quad [51]$$

By drawing perpendiculars from the vertices C and A to the opposite sides, we obtain, in a similar way:

$$\boldsymbol{b^2 = c^2 + a^2 - 2ca\cos B} \quad \ldots\ldots \quad [52]$$

$$\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos C} \quad \ldots\ldots \quad [53]$$

The relation expressed by [51], [52], [53], is often called the **Law of Cosines**, of which the following is the

Translation: *The square of any side of any triangle is equal to the sum of the squares of the other two sides, minus twice the product of these sides into the cosine of their included angle.*

While all other trigonometric relations of the sides and angles of plane triangles can be derived from the law of cosines, it is often more convenient to deduce them by independent methods.

97. Deduction of formulas by cyclical changes of the letters. Since a and A, b and B, or c and C, stand for *any* side of a triangle and the *opposite* angle, from any formula expressing a general relation between these parts another formula may be deduced by changing the letters in cyclical order. Thus, in [51] by changing the a to b, the b to c, the c to a, and the A to B, we obtain [52]; and in [52] by making the same or similar changes we obtain [53].

98. Relation of the three angles.

By Geometry, $A + B + C = 180°$, or $C = 180° - (A + B)$.

This relation is called the **Law of Angles.** It may be expressed by functions thus:

$$\cos C = \cos [180° - (A + B)] = -\cos (A + B). \qquad \text{Art. 85}$$

$$\therefore \text{ by [23], } \mathbf{\cos C = -\cos A \cos B + \sin A \sin B} \quad . \ . \ . \quad [54]$$

99. Relation of the sides to their opposite angles.

From Figs. 45, 46, we have

$$\frac{y}{c} = \sin A, \text{ or } y = c \sin A,$$

$$\frac{y}{a} = \sin C, \text{ or } y = a \sin C.$$

$$\therefore \mathbf{a \sin C = c \sin A} \quad . \ . \ . \ . \ . \ . \ . \quad [55]$$

Cyclically, Art. 97,

$$\mathbf{b \sin A = a \sin B} \quad . \ . \ . \ . \ . \ . \ . \quad [56]$$

$$\mathbf{c \sin B = b \sin C} \quad . \ . \ . \ . \ . \ . \ . \quad [57]$$

This relation may be deduced from [51] and [52] by eliminating c, which may be done thus:

[51] + [52], $c = a \cos B + b \cos A$ . . . . (f)

[51] − [52], $a^2 - b^2 = (a \cos B - b \cos A)c$ . . . . (g)

(f) × (g), $a^2 - b^2 = a^2 \cos^2 B - b^2 \cos^2 A$,

or $a^2(1 - \cos^2 B) = b^2(1 - \cos^2 A)$.

Whence, [11], $a \sin B = b \sin A$.

Formula [55], [56], or [57] may be expressed under the form of a proportion, as $a : b :: \sin A : \sin B$, which relation is called the **Law of Signs,** of which the following is the

Translation: *Any two sides of any triangle are proportional to the sines of their opposite angles.*

100. Relations of the sums and differences of two sides and their opposite angles.

From [56], $$\frac{a}{b} = \frac{\sin A}{\sin B}.$$

By composition and division,

$$\frac{a+b}{a-b} = \frac{\sin A + \sin B}{\sin A - \sin B}.$$

But by [45], $$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}.$$

$$\therefore \frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)} \quad . \quad . \quad . \quad . \quad . \quad . \quad [58]$$

By cyclical deduction (Art. 97) we may obtain similar expressions for the other pairs of sides.

This relation [58] is called the **Law of Tangents.**

Translation: *The sum of any two sides of any triangle is to their difference as the tangent of half the sum of the opposite angles is to the tangent of half their difference.*

101. Relation of the half of one angle to the three sides.

For brevity, let $a + b + c = 2s$; then $a + b - c = 2(s - c)$, $a - b + c = 2(s - b)$, and $-a + b + c = 2(s - a)$.

From (*c*) and (*d*), Art. 91, we have

$$2 \sin^2 \tfrac{1}{2} A = 1 - \cos A, \quad \text{and} \quad 2 \cos^2 \tfrac{1}{2} A = 1 + \cos A.$$

Solving [51] for $\cos A$,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

Substituting this value for $\cos A$ in the above, and we have

$$(1) \qquad 2 \sin^2 \tfrac{1}{2} A = \frac{2bc - b^2 - c^2 + a^2}{2bc} = \frac{a^2 - (b-c)^2}{2bc}$$

$$= \frac{(a + b - c)(a - b + c)}{2bc} = \frac{2(s-b)(s-c)}{bc};$$

(2) $$2\cos^2\tfrac{1}{2}A = \frac{2bc + b^2 + c^2 - a^2}{2bc} = \frac{(b+c)^2 - a^2}{2bc}$$
$$= \frac{(b+c+a)(b+c-a)}{2bc} = \frac{2s(s-a)}{bc}.$$

$$\therefore\ 2\sin^2\tfrac{1}{2}A = \frac{2(s-b)(s-c)}{bc},\ \text{and}\ 2\cos^2\tfrac{1}{2}A = \frac{2s(s-a)}{bc}.$$

Whence $$\sin\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}} \quad . \ . \ . \ . \ . \ . \quad [59]$$

$$\cos\tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}} \quad . \ . \ . \ . \ . \ . \ . \ . \quad [60]$$

and, Art. 70, $$\tan\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \quad . \ . \ . \ . \ . \ . \quad [61]$$

Cyclically,

$$\sin\tfrac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}} \ . \ [62], \qquad \sin\tfrac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{ab}} \ . \ [65]$$

$$\cos\tfrac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}} \ . \ . \ [63], \qquad \cos\tfrac{1}{2}C = \sqrt{\frac{s(s-c)}{ab}} \ . \ . \ [66]$$

$$\tan\tfrac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} \ . \ [64], \qquad \tan\tfrac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} \ . \ [67]$$

102. To find the area of any plane triangle.

I. *When the base and altitude are given.*

Let *ABC*, Art. 96, be the triangle; then (Art. 51),

$$K = \tfrac{1}{2}by \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad [68]$$

II. *When two sides and their included angle are given.*

Let b, c, be the sides and A the included angle; then $y = c\sin A$. Substituting in [68],

$$K = \tfrac{1}{2}bc\sin A \quad . \ . \ . \ . \ . \ . \ . \ . \quad [69]$$

III. *When the three sides are given.*

By [38], $\sin A = 2\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A$; hence from [69],

$$K = bc\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A.$$

In this, for $\sin\tfrac{1}{2}A$ and $\cos\tfrac{1}{2}A$, substitute their values from [59] and [60],

$$K = \sqrt{s(s-a)(s-b)(s-c)} \quad . \ . \ . \ . \ . \ . \quad [70]$$

103. To find the radius of a circle circumscribed about a triangle.

Let O be the centre of the circle circumscribed about the $\triangle ABC$, and R its radius.

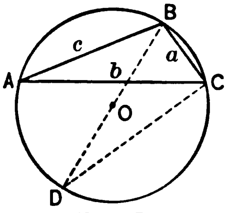

FIG. 47.

Through O draw the diameter BD and join CD; then $\angle DCB = 90°$, and

$$\angle CDB = \angle CAB = \angle A.$$

$$\therefore\ CB = BD \sin A, \text{ or } a = 2R \sin A.$$

Whence, $$R = \frac{a}{2 \sin A}.$$

Again, from [69], $$\sin A = \frac{2K}{bc},$$

$$\therefore\ R = \frac{abc}{4K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [71]$$

104. To find the radius of a circle inscribed in a triangle.

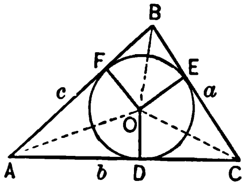

FIG. 48.

Let $r (= OD = OE = OF)$ be the radius.

$$\triangle ABC = \triangle OAC + \triangle OCB + \triangle OBA$$
$$= \frac{AC}{2} \times r + \frac{CB}{2} \times r + \frac{BA}{2} \times r$$
$$= \tfrac{1}{2}(AC + CB + BA) r = sr.$$

But $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$,

$$\therefore\ sr = \sqrt{s(s-a)(s-b)(s-c)},$$

whence, $$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}} \quad . \quad . \quad . \quad . \quad . \quad [72]$$

COR. I. Dividing [61] by [72],

$$\tan \tfrac{1}{2} A = \frac{r}{s-a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Cyclically, $$\tan \tfrac{1}{2} B = \frac{r}{s-b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

$$\tan \tfrac{1}{2} C = \frac{r}{s-c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

SOLUTION OF OBLIQUE TRIANGLES

105. Solvable Triangles. That any plane triangle may be solved when any three of its parts, except the three angles, are given, will be shown by considering each case that may arise and pointing out the means and method of solution. The three given parts may be:

I. *One side and two angles.*

II. *Two sides and the angle opposite to one of them.*

III. *Two sides and their included angle.*

IV. *The three sides.*

106. I. **When one side and two angles are given.**

First, we find the third angle by subtracting the sum of the given angles from 180°. We then find the required sides by the law of sines, as illustrated in the solution of the following example.

(1) Given $a = 804$, $A = 99° 55'$, $B = 45° 1'$; find C, b, c.

STATEMENT

$$C = 180° - (A + B).$$

By [56], $b \sin A = a \sin B$;

$$\therefore\ b = \frac{a}{\sin A} \times \sin B.$$

By [55], $c \sin A = a \sin C$;

$$\therefore\ c = \frac{a}{\sin A} \times \sin C.$$

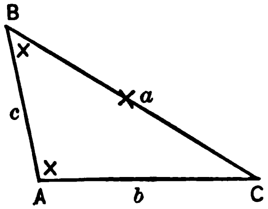

FIG. 49.

COMPUTATION BY NATURAL FUNCTIONS

$$C = 180° - (99° 55' + 45° 1') = 35° 4'.$$

$$b = \frac{804}{.98506} \times .70731 = 816.194 \times .70731 = 577.30.$$

$$c = \frac{804}{.98506} \times .57453 = 816.194 \times .57453 = 468.93.$$

NOTE. — $\sin 99° 55' = \sin (180° - 99° 55') \doteq \sin 80° 5'$.

BY LOGARITHMS

$$\log b = \log a + \operatorname{colog} \sin A + \log \sin B.$$

$$\begin{aligned} \log a &= 2.90526 \\ \operatorname{colog} \sin A &= 0.00654 \\ \log \sin B &= \underline{9.84961 - 10} \\ \log b &= 2.76141 \\ b &= 577.31. \end{aligned}$$

$$\log c = \log a + \operatorname{colog} \sin A + \log \sin C.$$

$$\begin{aligned} \log a &= 2.90526 \\ \operatorname{colog} \sin A &= 0.00654 \\ \log \sin C &= \underline{9.75931 - 10} \\ \log c &= 2.67111 \\ c &= 468.93. \end{aligned}$$

107. Exercise XV.

1. Given $b = 30.36$, $A = 103° 36'$, $B = 19° 21'$; find $C = 57° 3'$, $a = 89.06$, $c = 76.89$.
2. Given $a = 180$, $A = 38°$, $B = 75° 43'$; find $b = 283.33$, $c = 267.68$.
3. Given $a = 550$, $A = 10° 12'$, $B = 46° 36'$; find $b = 2256.6$, $c = 2598.9$.
4. Given $c = 93$, $C = 60° 5'$, $B = 63° 7'$; find $a = 89.782$, $b = 95.701$.
5. Given $b = 67.85$, $B = 13° 57'$, $C = 57° 13'$; find $a = 266.4$, $c = 236.6$.
6. Given $c = 998$, $A = 37° 58'$, $B = 65° 2'$; find C, a, b.
7. Given $a = 4.37$, $B = 48° 30'$, $C = 72° 8'$; find A, b, c.
8. Given $b = 12.4$, $A = 65° 15'$, $B = 79° 3'$; find C, a, c.
9. Given $c = .9310$, $A = 43° 6.4'$, $C = 100° 54.2'$; find B, a, b.

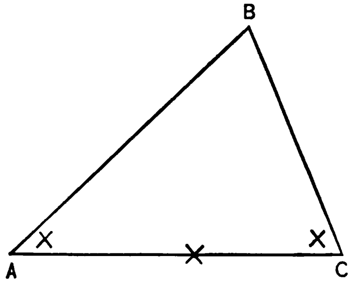

FIG. 50.

10. Two ships (A, C) are 1005 yd. apart (AC), and it is desired to know how far each is from a fort (B). To this end the angles ACB and CAB are measured, and found to be $78° 19'$, $54° 27'$, respectively. What are the distances AB, CB?

11. The base of a triangle is 600 ft., and the angles at the base are $30°$, $120°$. Find the other sides and the altitude.

12. Two men 1000 yd. apart on a plain, and facing each other, find that the angles of elevation of a balloon in the same vertical plain with themselves are $53°$ and $79° 12'$, respectively. Find the distance from the balloon to each observer, and also the height of the balloon above the plain. *Ans.* Height, 1059 yd.

108. II. **When two sides and the angle opposite to one of them are given.**

Knowing one angle and its opposite side, we first find the angle opposite the other given side by the law of sines. Now, knowing two angles, we find the third by the law of angles, and then find the remaining unknown side by the law of sines.

(1) Given $a = 117$, $c = 216$, $A = 22° 37'$; find C, B, b.

STATEMENT

By [55], $a \sin C = c \sin A$, $\therefore \sin C = \frac{c \sin A}{a}$. $B = 180° - (A + C)$.

By [56], $b \sin A = a \sin B$, $\therefore b = \frac{a \sin B}{\sin A}$.

COMPUTATION

$$\begin{aligned}
\log\sin A &= 9.58497-10\\
\operatorname{colog} a &= 7.93181-10\\
\log c &= \underline{2.33445}\\
\log\sin C &= 9.85123-10\\
\therefore\ C &= 45^\circ\,13.9'.
\end{aligned}
\qquad\qquad
\begin{aligned}
\log a &= 2.06819\\
\operatorname{colog}\sin A &= 0.41503\\
\log\sin B &= \underline{9.96670-10}\\
\log b &= 2.44992\\
b &= 281.79.
\end{aligned}$$

Whence, $B = 112^\circ\,9.1'$.

But in finding an angle from its sine, as C in the above, it should be remembered that the supplement of an angle has the same sine as the angle. Hence, denoting the supplement of C by C', and the corresponding values of B and b by B' and b', we have

$$\begin{aligned}
C' &= 180^\circ - C = 134^\circ\,46.1',\\
B' &= 180^\circ - (A + C') = 22^\circ\,36.9',\\
b' &= \frac{a\sin B'}{\sin A}.
\end{aligned}
\qquad\qquad
\begin{aligned}
\log a &= 2.06819\\
\operatorname{colog}\sin A &= 0.41503\\
\log\sin B' &= \underline{9.58494-10}\\
\log b' &= 2.06816\\
\therefore\ b' &= 116.99.
\end{aligned}$$

Hence this problem admits of two solutions; that is, there are two triangles which satisfy the conditions. The construction of the triangle from the data will make this apparent.

Lay off $XAE = A = 22^\circ\,37'$, and on AE take AB equal to $c = 216$. With B as a centre, and a radius equal to $a = 117$, describe an arc cutting AX in C and C'. Then either of the triangles ABC or ABC' will fulfil the conditions. In the first $C = ACB = 45^\circ\,13.9'$, $b = AC = 281.79$, and in the second $C' = AC'B = 134^\circ\,46.1'$, $b' = AC' = 116.99$.

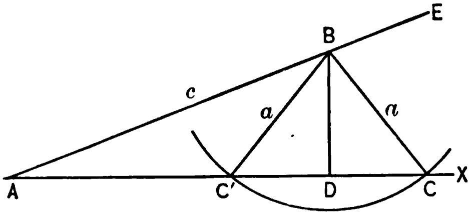

FIG. 51.

However, every problem under case II does not admit of two solutions; the data may be such that there may be *two* solutions, *one* solution, or *no* solution.

The conditions may be determined by examining the expression

$$\sin C = \frac{c \sin A}{a}.$$

(1) If $\frac{c \sin A}{a} = 1$, or $a = c \sin A (= BD)$, C is a right angle, and there is but one solution.

(2) If $\frac{c \sin A}{a} > 1$, or $a < c \sin A (= BD)$, C is an impossible angle, and there is no solution.

(3) If $\frac{c \sin A}{a} < 1$, or $a > c \sin A (= BD)$, C may have two values, one less than 90° and the other greater. If a is greater than c, then A must be greater than C, and therefore C must be an acute angle. That is, the smaller value only of C is admissible, and there is but one solution. If a is less than c, then either value may be taken for C, and there are two solutions.

109. General Solution. Let us find the value of b in terms of a, c, A, from the general relation

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

Transpose, $b^2 - 2(c \cos A)b = a^2 - c^2.$

Solve for b, $b = c \cos A \pm \sqrt{a^2 - c^2 \sin^2 A}.$

By this result we can compute the value of b by natural functions. By it we observe

(1) In general, b will have two values, which shows that there will be two solutions.

(2) When $a < c \sin A$, the values of b will be imaginary, which shows that there will be no solution.

(3) When $a = c \sin A$, $b = c \cos A$, which shows that C is a right angle and that there will be but one solution.

(4) When $a > c$, only one of the values of b will be positive, which shows that there will be only one solution.

(5) When $a < c$ and $> c \sin A$, there will be two solutions.

110. Exercise XVI.

1. Given $a = 4.536$, $c = 4.254$, $A = 37° 9'$; find $C = 34° 30'$, $b = 7.129$.

   NOTE.—Since $a > c$, there is but one solution.

2. Given $a = 22.829$, $c = 50$, $A = 27° 10'$; find $C = 90°$, $b = ?$

   NOTE.—Since $a = c \sin A$, there is but one solution, and $C = 90°$.

3. Given $a = 27.89$, $b = 22.71$, $B = 65° 38'$; find A, c.

   NOTE.—Since $b < a \sin B$, there is no solution.

4. Given $c = 18$, $b = 20$, $C = 55° 24'$; find $B = 66° 9'$, $a = 18.64$ and $B' = 113° 51'$, $a' = 4.08$.

NOTE. — Since $c < b$ and $c > b \sin C$, there are two solutions.

5. Given $a=168$, $b=97$, $A=21° 31'$; find $B=12° 14'$, $c = 254.4$.

6. Given $a = 51$, $b = 33$, $B = 30° 20'$; find $A = 51° 18'$, $c = 64.6$, and $A' = 128° 42'$, $c' = 23.4$.

7. Given $a=84$, $b=69$, $B=61° 47'$; show that A is impossible.

8. Given $a = 10.3572$, $c = 12$, $A = 59° 40'$; find $C = 90°$, $b = 6.06$.

9. Given $b = 4.521$, $c = 5.03$, $B = 40° 32.1'$; find $C = 46° 19'$, $a = 6.946$, and $C' = 133° 41'$, $a' = .7$.

10. Given $a = 186.8$, $b = 394.2$, $B = 114° 29.8'$; find A, c.

11. Given $a = 4.4$, $b = 5.21$, $A = 57° 37' 17''$; find B, c.

12. Given $a = 8.716$, $b = 9.787$, $A = 38° 14' 12''$; find B, c.

13. A hill stands on a horizontal plane, and wishing to know the distance of a point (A) in the plane from the top (B) of the hill, I select a point (C) at the foot of the hill, in the same vertical plane with B and A, and find that $BC = 910.6$ yd., $AC = 770.4$ yd., $CAB = 51° 9.1'$. Required the distance AB. *Ans.* 1168.2 yd.

14. One side of a parallelogram is 40, one of the diagonals 72, and the angle between the two diagonals 158° 23.5'. Find the other diagonal. *Ans.* 142.42.

111. III. **When two sides and their included angle are given.**

By using natural functions, we may first find the unknown side by the law of cosines, and then find the two unknown angles by the law of sines.

To use logarithms, we first find the two unknown angles by the law of tangents, and then find the unknown side by the law of sines.

(1) Given $a = 17$, $b = 12$, $C = 59° 17'$; find A, B, c.

FIRST STATEMENT

By [53], $c^2 = a^2 + b^2 - 2ab \cos C$, $\therefore\ c = \sqrt{a^2 + b^2 - 2ab \cos C}$.

By [55], $c \sin A = a \sin C$, $\therefore\ \sin A = \dfrac{a \sin C}{c}$.

$$B = 180° - (A + C).$$

COMPUTATION BY NATURAL FUNCTIONS

$$c = \sqrt{17^2 + 12^2 - 2 \times 17 \times 12 \times .51079} = \sqrt{224.59768} = 14.9866.$$

$$\sin A = \frac{17 \times .8597}{14.9866} = .97520. \quad \therefore\ A = 77^\circ\,12.8'.$$

$$B = 180^\circ - (59^\circ\,17' + 77^\circ\,12.8') = 43^\circ\,30.2'.$$

SECOND STATEMENT

$$A + B = 180^\circ - C, \qquad \therefore\ \tfrac{1}{2}(A + B) = 90^\circ - \tfrac{1}{2}C.$$

From [58], $\tan\frac{1}{2}(A - B) = \dfrac{(a - b)\tan\frac{1}{2}(A + B)}{(a + b)}.$

By this, we find $\frac{1}{2}(A-B)$. Now, knowing $\frac{1}{2}(A+B)$ and $\frac{1}{2}(A-B)$, we add their values to get A, and subtract them to get B, since

$$A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B), \text{ and } B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B).$$

By [57], $c \sin B = b \sin C, \quad \therefore\ c = \dfrac{b \sin C}{\sin B}.$

SOLUTION BY LOGARITHMS

$$\begin{array}{rlr}
a + b = 29 & \quad a - b = 5 & \tfrac{1}{2}(A + B) = 60^\circ\,21.5' \\
\log(a - b) = & .69897 & \log b = 1.07918 \\
\operatorname{colog}(a + b) = & 8.53760 - 10 & \operatorname{colog}\sin B = 0.16216 \\
\log\tan\tfrac{1}{2}(A + B) = & \underline{10.24485 - 10} & \log\sin C = \underline{9.93435 - 10} \\
\log\tan\tfrac{1}{2}(A - B) = & 9.48142 - 10 & \log c = 1.17569 \\
\tfrac{1}{2}(A - B) = & 16^\circ\,51.3'. & c = 14.986.
\end{array}$$

$$\therefore\ A = 77^\circ\,12.8',\ B = 43^\circ\,30.2'$$

112. Exercise XVII.

1. Given $b = 14.367$, $c = 11.412$, $A = 42^\circ\,14.6'$; find $C = 52^\circ\,21'$, $a = 9.69$.
2. Given $a = 729.6$, $b = 613.8$, $C = 32^\circ\,12'$; find A, B, c.
3. Given $a = 101.47$, $c = 99.367$, $B = 47^\circ\,48.2'$; find $A = 67^\circ\,27'$, $C = 64^\circ\,45'$, $b = 81.4$.
4. Given $b = 99.685$, $c = 312.375$, $A = 130^\circ\,9.4'$; find $B = 11^\circ\,26'$, $a = 384.3$.
5. Given $b = 3000.9$, $c = 1587.2$, $A = 86^\circ\,4'$; find B, C, a.
6. Given $a = 27.43$, $c = 22.428$, $B = 15^\circ\,22.6'$; find $A = 118^\circ\,56'$, $b = 8.31$.

7. In Fig. 46, if $b = 7$, $c = 7$, and $\angle BAD = 120°$, what is the value of a? *Ans.* 12.124.

8. Two sides of a triangle are as 5 to 6, and the included angle is 15° 28′. Find the other two angles. *Ans.* 116° 4′; 48° 28′.

9. Two sides of a triangle are 10 and 11, and the included angle is 50°. Find the third side.

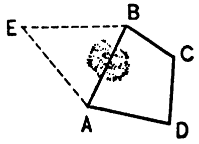

FIG. 52.

10. In surveying a tract of land *ADCB*, I found it impracticable to measure the side *AB* on account of thick brushwood lying between *A* and *B*. I therefore measured *AE*, 9.17 ch., and *EB*, 3.12 ch., and found the angle $AEB = 33° 7.1'$. Required the distance from *A* to *B*. *Ans.* 6.77 ch.

113. IV. **When the three sides are given.**

By using natural functions we may find the three angles by the law of cosines, or we may find the first angle by the law of cosines, the second by the law of sines, and the third by the law of angles.

To use logarithms we find two of the angles by the formulas under Art. 101, and the third angle by the law of angles. In selecting the formulas from Art. 101, the formula for cos should not be taken if half the angle is near 0°, nor the one for sin if half the angle is near 90°. In general, the formulas for the tan are to be preferred.

If all the angles are to be found, it will be most convenient to first find the radius of the inscribed circle by [72], and then find *A*, *B*, *C* by (*a*), (*b*), (*c*), Art. 104.

(1) Given $a = 19$, $b = 34$, $c = 49$; find *A*, *B*, *C*.

STATEMENT I

By [52], $b^2 = a^2 + c^2 - 2ac\cos B$, $\therefore \cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$.

By [56], $b\sin A = a\sin B$, $\therefore \sin A = \dfrac{a \times \sin B}{b}$.

$$C = 180° - (A + B).$$

COMPUTATION BY NATURAL FUNCTIONS

$$\cos B = \frac{19^2 + 49^2 - 34^2}{2 \times 19 \times 49} = .86251, \quad \therefore B = 30° 24'.$$

$$\sin A = \frac{19 \times .50603}{34} = .28278, \quad \therefore A = 16° 25.6'.$$

$$C = 180° - (30° 24' + 16° 25.6') = 133° 10.4'.$$

STATEMENT II

By [60], $\cos\frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$. By [63], $\cos\frac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}}$.

$$C = 180^\circ - (A + B).$$

SOLUTION BY LOGARITHMS

$$\log\cos\tfrac{1}{2}A = \tfrac{1}{2}(\log s + \log(s-a) + \operatorname{colog} b + \operatorname{colog} c),$$

$$\log\cos\tfrac{1}{2}B = \tfrac{1}{2}(\log s + \log(s-b) + \operatorname{colog} a + \operatorname{colog} c).$$

$$\begin{array}{rl rl rl}
a = & 19 & \log s = & 1.70757 & \log s = & 1.70757 \\
b = & 34 & \log(s-a) = & 1.50515 & \log(s-b) = & 1.23045 \\
c = & \underline{49} & \operatorname{colog} b = & 8.46852 - 10 & \operatorname{colog} a = & 8.72125 - 10 \\
2s = & 102 & \operatorname{colog} c = & \underline{8.30980 - 10} & \operatorname{colog} c = & \underline{8.30980 - 10} \\
s = & 51, & 2) & \underline{19.99104 - 20} & 2) & \underline{19.96907 - 20} \\
s-a = & 32, & \log\cos\frac{1}{2}A = & 9.99552 - 10 & \log\cos\frac{1}{2}B = & 9.98453 - 10 \\
s-b = & 17. & \frac{1}{2}A = & 8^\circ 13', & \frac{1}{2}B = & 15^\circ 12', \\
 & & A = & 16^\circ 26'. & B = & 30^\circ 24'.
\end{array}$$

114. Exercise XVIII.

1. Given $a = 7$, $b = 8$, $c = 9$; find $A = 48^\circ 11'$, $C = 73^\circ 24'$.

2. Given $a = 3$, $b = 8$, $c = 9$; find $B = 61^\circ 13'$, $C = 99^\circ 36'$.

3. Given $a = 19$, $b = 34$, $c = 49$; find A, B, C, and check by the law of angles.

4. Given $a = 12$, $b = 16$, $c = 20$; find $A = 36^\circ 52'$, $C = 90^\circ$.

5. Given $a = 18$ $b = 18$, $c = 30$; find $A = B = 33^\circ 33'$, $C = 112^\circ 53'$.

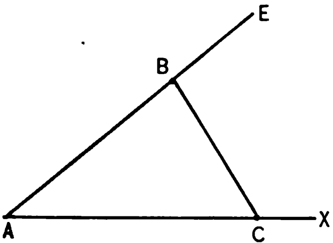

FIG. 53.

6. Given $a = b = c = 25$; find A, B, C.

7. Given $a = 63.89$, $b = 138.24$, $c = 121.15$; find $B = 91^\circ 18'$.

8. Desiring to know the size of the angle XAE, I lay off $AB = 26$ in., $AC = 31$ in., and then find that the distance from B to C is 21 in. Find the angle A. *Ans.* $42^\circ 6'$.

9. Two sides of a triangle are 51 and 65. What must their included angle be in order that the third side shall be 20?

AREA OF AN OBLIQUE TRIANGLE

115. The area of a triangle is found by Art. 102.

(1) Given $a = 97$, $b = 83$, $c = 71$; find the area K.

From [70], we have

$$2 \log K = (\log s) + \log (s - a) + \log (s - b) + \log (s - c).$$

$$\begin{array}{rrr}
a = \ \ 97 & s = 125.5, & \log s = 2.09864 \\
b = \ \ 83 & s - a = \ \ 28.5, & \log (s - a) = 1.45484 \\
c = \ \ \underline{71} & s - b = \ \ 42.5, & \log (s - b) = 1.62839 \\
2s = 251 & s - c = \ \ 54.5. & \log (s - c) = \underline{1.73640} \\
s = 125.5. & & 2)\underline{6.91827} \\
 & & \log K = 3.45913 \\
 & & \therefore \ K = 2878.3.
\end{array}$$

116. Exercise XIX.

1. Given $a = 10$, $b = 16$, $c = 12$; find $K = 59.92$.

2. Given $a = 7.9$, $b = 8.96$, $c = 10.46$; find $K = 34.4$.

3. Given $a = 46.7$, $b = 84.5$, $c = 75.6$; find $K = 1755.1$.

4. Given $a = 38.09$, $c = 11.2$, $B = 67° 55'$; find K.

5. Given $b = 12.5$, $c = 25$, $A = 38°$; find $K = 96.2$.

6. Given $a = 32.5$, $b = 56.3$, $C = 47° 5.5'$; find $K = 670$.

7. Two sides of a triangle are 34.56 ch. and 48.5 ch., and the included angle is $38° 45' 40''$. Find the area. *Ans.* 52.47 A.

8. Two angles of a triangle are $38° 18'$ and $91° 28'$, and the included side is 39.5 ch. Find the area. *Ans.* 62.88 A.

9. Two sides of a parallelogram are 59.8 ch. and 37.05 ch., and the included angle is $72° 10'$. Find the area.

10. In the quadrilateral $ABCD$, $AB = 30$ ch., $BC = 25$ ch., $CD = 37$ ch., $DA = 13$ ch., and the diagonal $AC = 40$ ch. Find the area. *Ans.* 61.45 A.

11. Show that the area of a quadrilateral is equal to one-half the product of its diagonals into the sine of their included angle.

12. Show that the area of a regular polygon of n sides inscribed in a circle whose radius is R, is one-half the product of n by the square of R into the sine of $\frac{1}{n}(360°)$.

MISCELLANEOUS EXERCISES

117. Exercise XX.

1. The three sides of a triangle are 8 ch., 9 ch., and 11 ch. Find the area. *Ans.* 3.55 acres.

2. Two sides of a triangle are 42 ch. and 51 ch., and their included angle is 33° 18′. Find the area.

3. The diagonals of a quadrilateral are 15 and 9.8, and cross each other at an angle of 138°. Find the area. *Ans.* 49.18.

4. In a field *ABCD*, the sides *AB*, *BC*, *CD*, *DA* are 31, 47.2, 50.4, 21 rd., respectively, and the diagonal *AC* is 62.2 rd. Find the area. *Ans.* 1192 sq. rd.

5. The area of a triangle is 54.24, and two of its sides are 10.6 and 13.8. Find the angle between these sides. *Ans.* 47° 52′.

6. The diagonals of a parallelogram are 81 and 106, and cross each other at an angle of 29° 18′. Find the sides, angles, and area of the parallelogram.

7. Two sides of a triangle are 112.46 and 99.54, and the difference of their opposite angles is 15° 48′ 32″. Find the third side. *Ans.* 86.02.

8. The parallel sides of a trapezoid are 57 and 109, and the angles at the extremities of the latter are 53° 49′ and 67° 55′. Find the non-parallel sides. *Ans.* 56.65; 49.35.

9. From the base of a wall the angle of elevation of the top of a pole is 23° 19′, and from a window 31.5 ft. above, it is 14° 13′. Find the height of the pole. *Ans.* 76.42 ft.

10. From a ship the horizontal angle subtended by a round fort is 8° 20′, but if the ship were to go 680 ft. nearer to the fort, it would be 10° 12′. Find the distance of the ship from the fort.

11. From a point at the foot of a mountain, the elevation of the top is 60°. After going 1760 yd. towards the top up a plane making an angle of 30° with the horizon, the angle of elevation of the top is 75°. Find the height of the mountain. *Ans.* 4164.2 yd.

12. A person goes 84 yd. up a slope of 2 feet in 7 from the edge of a river, and observes that the angle of depression of the edge of the water on the opposite side is $2\frac{1}{4}$°. Find the width of the river. *Ans.* 530.53 yd.

13. The sides of a triangle are 17, 21, 28. Find the two angles into which the largest angle of the triangle is divided by drawing a line from its vertex to the middle point of the opposite side.

14. To find the distance between two inaccessible points, A and B, a line CD, crossing AB, is measured, its length being 600 ft. The angle $DCB = 58° 12'$, $CDB = 49° 38'$, $ACD = 74° 16'$, $ADC = 62° 13'$. Find AB. *Ans.* 1151 ft.

15. One side and the opposite angle of a triangle are 309 and $21° 14.4'$; if one of the remaining sides is 360, what is the other? *Ans.* 615.7 or 55.41.

16. Wishing to find the "high water width" (AB) of the Mississippi River directly in front of the Louisiana State University, that is, the distance from the railroad (A) on one side to the levee (B) on the other, the following measurements were made: a line $AC = 1000$ ft. along the railroad track, $\angle BAC = 94°$, $\angle BCA = 70° 41.6'$. Required the width AB. *Ans.* 3575 ft.

17. The "low water width" (BC) of the Mississippi River at the same point as in Ex. 16, is 2950 ft., and from a point (A) of the university campus overlooking the river, the angles of depression of the nearer (B) and further (C) edges of the water are $7° 49.8'$ and $56.4'$. Required the height of the university campus above the "low water." *Ans.* 54.96 ft.

18. In measuring a line from A to B, whose direction was N. 40° E., it was found to cross a marsh, say M. To pass around M, a distance $CD = 144.31$ ch. was measured, starting from a point C of AB, and making with AB an angle $BCD = 19° 54.4'$. At D it was found that a line DE, making an angle $CDE = 140° 10.3'$ with CD, would cross the line AB at some point, say G, beyond M. Find the distances DG and CG, and also the angle DGB in order that the course of AB may be resumed.

Ans. $DG = 143.98$ ch.; $CG = 271.06$ ch.; $DGB = 160° 3.7'$.

19. When the elevation of the sun is 48°, a pole, standing vertically on a slope whose angle with the horizon is 15°, casts a shadow directly down the slope 44.3 ft. long. How high is the pole? *Ans.* 30.06 ft.

20. Three circles whose radii are 12, 16, and 25 are tangent externally. Find the angles between the lines joining their centres.

21. The three points A, B, C are in a horizontal plane, and CD is a vertical pole; $AB = 435.53$ ft., $CAB = 140° 40.2'$, $CBA = 10° 7.6'$, $CAD = 32° 45.6'$, $CBD = 10° 7.3'$. Find the height (CD) of the pole.

Ans. 101 ft.

22. Between the points A and B, a distance of 1¾ miles, the grade of a road on a straight line would be 1 foot in 40. What would be the length of a zigzag road from A to B that would have a grade of 1 foot in 70?

23. An engineer, desiring to know the distance between two hostile forts (A, B) on the same horizontal plane, goes up in a balloon to the height of 517.3 rd., and from that point (C) observes that the angles of depression of A and B are $21° 9' 18''$ and $23° 15' 34''$, and that the angle $ACB = 15° 13' 15''$. Find the distance between the forts.

Ans. 383.35 rd.

24. The four sides of a city block are $AB = 423.24$, $BC = 162.36$, $CD = 420.80$, and $DA = 160.60$ ft., the first two sides being perpendicular to each other. Find the angles between the other sides.

25. On the bank of a river there is a column 200 ft. high supporting a statue 30 ft. high; the statue to an observer on the opposite bank subtends an equal angle with a man 6 ft. high standing at the base of the column. Required the width of the river.

Ans. $10\sqrt{115}$ ft.

Note. — After the student has learned to compute readily by natural functions and logarithms, it will be sufficient to give only the statements in solving problems like the two following.

26. Wishing to determine the distance between two objects (A, B) on the opposite side of a river, I lay off a line, $CD = 800$ ft., along the river, C being nearly opposite A; and then find by measurement that $ACB = 57° 20'$, $ACD = 94° 20'$, $ADB = 53° 20'$, $BDC = 98° 35'$. Find the required distance AB.

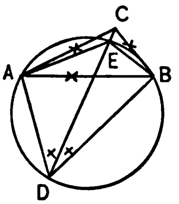

FIG. 54.

27. Given $AB = 800$ yd., $AC = 600$ yd., $BC = 400$ yd., $ADC = 33° 45'$, $BDC = 22° 30'$. Required AD, CD, BD.

$AD = 710.15$ yd., $CD = 1042.5$ yd., $BD = 934.28$ yd.

Hint. — Describe the circumference through A, B, D, and draw AE, BE; then $EAB = BDC$, and $EBA = ADC$. Find $\angle ABC$, Art. 113, then $\angle EBC$, then EB, then $\angle C$, Art. 111, etc.

PART TWO

SPHERICAL TRIGONOMETRY

CHAPTER V

RIGHT SPHERICAL TRIANGLES

DEFINITIONS AND ELEMENTARY PRINCIPLES

118. Spherical Triangle. Let *A, B, C,* be any three points on the surface of a sphere whose centre is *O.* Through (1) *O, A, B,* (2) *O, A, C,* (3) *O, B, C,* pass planes forming the triangular spherical pyramid whose vertex is *O,* and whose base is *ABC,* then the *figure* *ABC* is a spherical triangle, of which the three sides, *AB, AC, BC,* are arcs of great circles, and the three angles, *A, B, C,* are equal to the plane angles which measure the diedral angles of the pyramid.

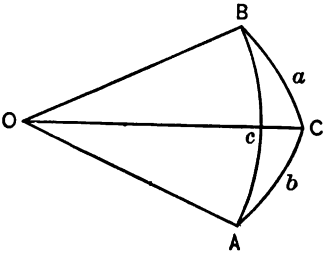

FIG. 55.

119. Spherical Trigonometry treats of the trigonometric relations among the sides and angles of a spherical triangle. These relations are not affected by varying the radius of the sphere; that is, they are independent of the radius. Hence, for simplicity, the radius, in this book, will always be considered unity or 1. Under this supposition the relative lengths of the sides *AB, BC, AC,* are the circular measures of the face angles of the pyramid *AOB, BOC, AOC,* respectively.

120. Notation. As in Plane Trigonometry, the angles of triangles will be denoted by *A, B, C,* and the opposite sides by *a, b, c.* The sides are usually expressed in degree measure; that is, *a, b, c,* stand for the numbers of degrees in the arcs *CB, AC, AB,* or the angles *COB, AOC, AOB.*

121. Parts. Every spherical triangle has six parts, — three sides and three angles. Any two parts are said to be of the same or different *species* according (1) as they are both less or greater than 90°, or (2) as the one is less and the other greater than 90°.

122. A spherical triangle is called **trirectangular** when it has three right angles, and **birectangular** when it has two right angles.

123. Geometric Properties. This book treats only of spherical triangles whose parts are each less than 180°. Of these the following properties are proved in Geometry:

1. The sum of any two sides of a spherical triangle is greater than the third side.

2. In any spherical triangle, the greater side lies opposite the greater angle; and, conversely, the greater angle lies opposite the greater side.

3. The sum of the sides of a spherical triangle is less than 360°.

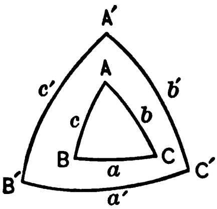

FIG. 56.

4. The sum of the angles of a spherical triangle is greater than 180°, and less than 540°.

5. If $A'B'C'$ is the polar triangle of ABC, that is, if A, B, and C are the poles of the sides $B'C'$, $C'A'$, and $A'B'$, respectively, then, conversely, ABC is the polar triangle of $A'B'C'$.

6. In two polar triangles, each angle of one is measured by the supplement of the side lying opposite the homologous angle of the other; that is,

$$a' = 180° - A. \qquad b' = 180° - B. \qquad c' = 180° - C.$$

$$A' = 180° - a. \qquad B' = 180° - b. \qquad C' = 180° - c.$$

ELEMENTARY FORMULAS

124. Let C be the right angle of the right spherical triangle ABC, O the centre of the sphere, and OA, OB, OC radii, each equal to 1.

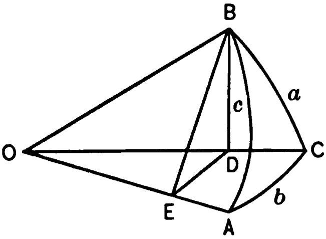

FIG. 57.

Draw $BD \perp OC$ and $BE \perp OA$; draw ED, then ED is $\perp$ BD and OA, since ED and DB are the projections of EB on the perpendicular planes OCA and OCB.

$$\therefore\ BD = \sin a, \quad OD = \cos a,$$

$$BE = \sin c, \quad OE = \cos c.$$

The right triangle OED gives

$OE = OD \cos b$; that is, $\qquad \mathbf{\cos c = \cos a \cos b}$. [73]

Since BE and DE are $\perp OA$, $\angle DEB = \angle A$; hence

(1) $\sin A = \dfrac{BD}{BE} = \dfrac{\sin a}{\sin c}$; $\quad\therefore\quad \boldsymbol{\sin a = \sin A \sin c}$. [74]

Similarly, $\quad \boldsymbol{\sin b = \sin B \sin c}$. [75]

(2) $\cos A = \dfrac{ED}{BE} = \dfrac{OE \tan b}{OE \tan c}$; $\quad\therefore\quad \boldsymbol{\cos A = \tan b \cot c}$. [76]

Similarly, $\quad \boldsymbol{\cos B = \tan a \cot c}$. [77]

(3) $\cot A = \dfrac{ED}{DB} = \dfrac{OD \sin b}{OD \tan a}$; $\quad\therefore\quad \boldsymbol{\sin b = \cot A \tan a}$. [78]

Similarly, $\quad \boldsymbol{\sin a = \cot B \tan b}$. [79]

[73] × [75] × [76],

$$\cos c \sin b \cos A = \cos a (\cos b \tan b)(\sin c \cot c) \sin B$$
$$= \cos a \sin b \cos c \sin B.$$

$\therefore\ \boldsymbol{\cos A = \sin B \cos a}$ . . . . [80]

Similarly, $\quad \boldsymbol{\cos B = \sin A \cos b}$ . . . . [81]

By [73], [80], [81], $\cos c = \cos a \cos b = \dfrac{\cos A}{\sin B} \times \dfrac{\cos B}{\sin A}$.

$\therefore\ \boldsymbol{\cos c = \cot A \cot B}$ . . . . [82]

125. Generality of the Formulas. The preceding ten formulas have been proved only for the case when the legs (a, b) are both $< 90°$. It therefore remains to be shown that they are also true (1) when one of the legs is $< 90°$ and the other $> 90°$, and (2) when both legs are $> 90°$.

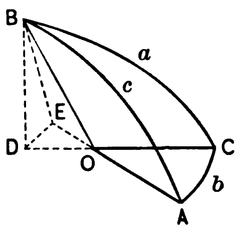

FIG. 58.

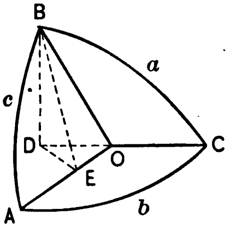

FIG. 59.

In Fig. 58, let $a > 90°$ and $b < 90°$, and in Fig. 59, let $a > 90°$ and $b > 90°$. In both figures, precisely as in Fig. 57, draw $BD \perp OC$ (produced) and $BE \perp OA$ (produced, if necessary), then draw ED. Just as in Art. 124, ED will be $\perp BD$ and OA, and therefore $BD = \sin a$, $OD = \cos a$, $BE = \sin c$, $OE = \cos c$, $\angle DEB = \angle A$. Consequently the proofs under Art. 124 apply, word for word, to each of these cases, and the ten formulas are therefore general.

126. Relations with Respect to Species. I. It is evident from [73] that $\cos c$ is *positive* when $\cos a$ and $\cos b$ have the *same* sign, and *negative* when they have *opposite* signs; therefore *the hypotenuse* (c) *of a right triangle is acute or* $< 90°$ *when the two legs* (a, b) *are of the same species, and obtuse or* $> 90°$ *when they are of different species.*

II. Since each part is $< 180°$, $\sin a$ is always positive. It is evident then from [79] that $\cot B$ or $\tan B$ and $\tan b$ always have the same sign; that is, *either oblique angle and its opposite side are always of the same species.*

127. Napier's Circular Parts and Rules. Formulas [73] to [82] are used in solving right spherical triangles, and the difficulty of remembering and producing the right formula for any given case is largely removed by the use of Napier's Circular Parts and Rules, which we will now explain.

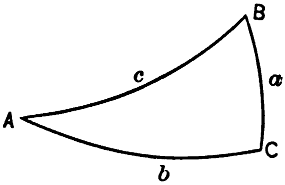

FIG. 60.

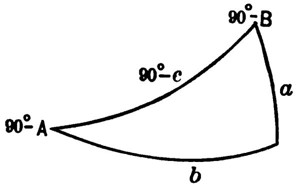

FIG. 61.

128. Circular Parts. Instead of writing the triangle in the usual way, as in Fig. 60, we write it as in Fig. 61; that is, instead of the hypotenuse and the two oblique angles their complements are employed, the right angle C being omitted. The five parts, a, b, $90° - A$, $90° - c$, $90° - B$, are *Napier's Circular Parts,* and with respect to any part, as $90° - A$, the parts standing next to it in the figure, $90° - c$ and b, are called the *adjacent parts,* and the other two parts, $90° - B$ and a, the *opposite parts.* With respect to the part $90° - c$, $90° - A$ and $90° - B$ are adjacent parts, and a and b opposite parts.

129. Rules. I. *The sine of any part is equal to the product of the tangents of the adjacent parts.*

II. *The sine of any part is equal to the product of the cosines of the opposite parts.*

NOTE.—These Rules may be more easily remembered by observing that in Rule I the descriptive terms *adjacent* and *tangents* contain the vowel *a*, and in Rule II the descriptive terms *opposite* and *cosines* contain the vowel *o*.

In illustration of these Rules let us, by them, produce formulas [73] to [82]. We may select any part to begin with, as a (Fig. 61), then by Rule

I. $\sin a = \tan(90^\circ - B)\tan b,$ or $\sin a = \cot B \tan b,$

II. $\sin a = \cos(90^\circ - A)\cos(90^\circ - c),$ or $\sin a = \sin A \sin c,$

which are formulas [79] and [74].

Again, beginning with $90^\circ - B$, we have by Rule

I. $\sin(90^\circ - B) = \tan a \tan(90^\circ - c),$ or $\cos B = \tan a \cot c,$

II. $\sin(90^\circ - B) = \cos b \cos(90^\circ - A),$ or $\cos B = \cos b \sin A,$

which are formulas [77] and [81].

Let the student in a similar manner produce the other six formulas of Art. 124.

That the formulas thus obtained by the Rules are the same as those obtained in Art. 124, is a proof of the correctness of the Rules.

SOLUTION OF RIGHT SPHERICAL TRIANGLES

130. Solvable Cases. It is possible to solve, by means of formulas [73] to [82], any right spherical triangle when two of its parts, not including the right angle, are given.

131. Method without Napier's Rules. Teachers of Trigonometry differ as to the advisability of using Napier's Rules. Some think it preferable to remember the formulas by their analogy to the corresponding ones for right plane triangles. This analogy is very striking when presented as follows:

Denote the corresponding sides and angles of a plane right triangle also by $a, b, c, A, B, C,$ and we have

For Plane △	For Spherical △	
$\sin A = \frac{a}{c}.$	$\sin A = \frac{\sin a}{\sin c}$ . . .	[83]
$\cos A = \frac{b}{c}.$	$\cos A = \frac{\tan b}{\tan c}$ . . .	[84]
$\tan A = \frac{a}{b}.$	$\tan A = \frac{\tan a}{\sin b}$ . . .	[85]
$\cos A = \sin B.$	$\cos A = \sin B \cos a$ . .	[86]
$\cot A \cot B = 1.$	$\cot A \cot B = \cos c$ . . .	[87]
$c^2 = a^2 + b^2.$	$\cos c = \cos a \cos b$ . .	[88]

These six formulas, including the four which may be derived from [83], [84], [85], [86] by changing *A*, *B*, *a*, *b*, in cyclical order, are the same as formulas [73] to [82].

To solve a triangle without the use of Napier's Rules, we take from formulas [83] to [88], in order, those which involve the given parts and the required part, remembering that [83] to [86], by cyclical changes, can be used for *B* as well as *A*.

132. Method by Napier's Rules. Draw a figure like Fig. 61, and mark the given parts, as in the following example. There is always one of the unknown parts which is either adjacent or opposite to the two given parts, and this is the part to be found first; and it is found by Rule I or II, according as the given parts are adjacent or opposite to it.

We proceed in a similar manner to find the other parts, though with greater facility, since we have more known parts to take as the adjacent or opposite parts.

133. About the Signs. Careful attention must be given to the algebraic signs of the functions, remembering that the cos, tan, cot of an angle between 90° and 180° is negative. The signs of the required functions can usually be determined by the formulas on the principle that the product or quotient (1) of like signs is +, and (2) of unlike signs is −. In computations by logarithms the signs of the functions need not be carried along with the work, but the proper sign, as determined by the formulas, must be prefixed to the result.

If a given cos, tan, or cot is (1) +, the corresponding angle is the acute angle given by the tables; (2) if −, the corresponding angle is the supplement of the acute angle obtained from the tables.

In finding the angle corresponding to a given positive sine, since the sine of an angle is the same as the sine of its supplement, both the acute angle obtained from the tables and its supplement must be retained as solutions, unless one or the other is seen to be inconsistent with the principles of Art. 126.

134. Number of Cases. The two given parts may be (1) the two legs; (2) the hypotenuse and one leg; (3) one leg and the opposite angle; (4) one leg and the adjacent acute angle; (5) the hypotenuse and one acute angle; or (6) the two acute angles. These several cases are solved in a similar manner, and two or three examples will serve to illustrate the process.

NOTE.—In the solution of the following illustrative examples, the formulas in the second statement are derived by means of Napier's Rules. The teacher can use whichever method he prefers. The Author has usually employed Napier's Rules; they are thought to expedite the work of the student, and aid him in retaining the formulas.

135. Illustrative Examples.

(1) Given $a = 132° \, 6'$, $b = 77° \, 51'$; find A, B, c.

First Statement	Second Statement
By [88], $\cos c = \cos a \cos b$.	By II, $\cos c = \cos a \cos b$.
By [85], $\tan A = \tan a \div \sin b$.	By I, $\cos A = \tan b \cot c$.
By [85], $\tan B = \tan b \div \sin a$.	By I, $\cos B = \tan a \cot c$.

Check: By [87], $\cot A = \cos c \tan B$, or $\tan A \tan B \cos c = 1$.

NOTE. — Of the required parts, there are two ways of finding their quadrants: (1) By Art. 126, since $a > 90°$, A must be $> 90°$; since $b < 90°$, B must be $< 90°$; and since a and b are of different species, c must be $> 90°$. (2) Referring to the first statement, since $\cos a$ is $-$ and $\cos b$ is $+$, $\cos c$ must be $-$, and therefore $c > 90°$; since $\tan a$ is $-$ and $\sin b$ is $+$, $\tan A$ must be $-$, and therefore $A > 90°$; and since $\tan b$ is $+$ and $\sin a$ is $+$, $\tan B$ must be $+$, and therefore $B < 90°$.

COMPUTATION

$\log \cos a = 9.82635 - 10$	$\log \tan a = 10.04404 - 10$
$\log \cos b = 9.32319 - 10$	$\text{colog} \sin b = 0.00984$
$\log \cos c = 9.14954 - 10$	$\log \tan A = 10.05388 - 10$
$180° - c = 81° \, 53.3'$,	$180° - A = 48° \, 32.7'$,
$c = 98° \, 6.7'$.	$A = 131° \, 27.3'$.

$\log \tan b = 10.66697 - 10$	$\log \cos c = 9.14954 - 10$
$\text{colog} \sin a = 0.12961$	$\log \tan A = 10.05388 - 10$
$\log \tan B = 10.79658 - 10$	$\log \tan B = 10.79658 - 10$
$B = 80° \, 55.4'$.	$\log 1 = .00000$.

(2) Given $a = 170° \, 38'$, $B = 100° \, 40'$; find A, b, c.

First Statement	Second Statement
By [86], $\cos A = \cos a \sin B$.	By II, $\cos A = \cos a \sin B$.
By [85], $\tan b = \sin a \tan B$.	By I, $\sin b = \tan a \cot A$.
By [84], $\tan c = \tan a \div \cos B$.	By II, $\cos c = \cos a \cos b$.

Check: By [84], $\cos A = \tan b \div \tan c$.

NOTE. — The quadrants in which A, b, c lie may be determined as in the preceding examples. It is sufficient to note that, since $\cos A$ and $\tan b$ are $-$, in finding the angles corresponding to these, the supplements of the acute angles obtained from the tables must be taken.

COMPUTATION

$$\begin{aligned}
\log\cos a &= 9.99417-10\\
\log\sin B &= \underline{9.99243-10}\\
\log\cos A &= 9.98660-10\\
180^\circ - A &= 14^\circ\ 10',\\
A &= 165^\circ\ 50'.
\end{aligned}$$

$$\begin{aligned}
\log\tan a &= 9.21735-10\\
\operatorname{colog}\cos B &= \underline{0.73261}\\
\log\tan c &= 9.94996-10\\
c &= 41^\circ\ 42.4'.
\end{aligned}$$

$$\begin{aligned}
\log\sin a &= 9.21152-10\\
\log\tan B &= \underline{10.72504-10}\\
\log\tan b &= 9.93656-10\\
180^\circ - b &= 40^\circ\ 50',\\
b &= 139^\circ\ 10'.
\end{aligned}$$

Check

$$\begin{aligned}
\log\tan b &= 9.93656-10\\
\operatorname{colog}\tan c &= \underline{0.05004}\\
\log\cos A &= 9.98660-10
\end{aligned}$$

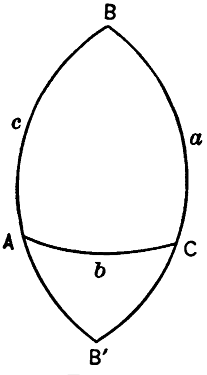

FIG. 62.

(3) Given $b = 68^\circ\ 35'$, $B = 82^\circ\ 25'$; find a, c, A.

FIRST STATEMENT

By [85], $\sin a = \tan b \div \tan B$.

By [86], $\sin A = \cos B \div \cos b$.

By [83], $\sin c = \sin b \div \sin B$.

SECOND STATEMENT

By I, $\sin a = \tan b \cot B$.

By II, $\cos A = \cos a \sin B$.

By I, $\cos c = \cos a \cos b$.

Check: By [83], $\sin A = \sin a \div \sin c$.

NOTE. — In cases of this kind, *where a leg and its opposite angle are given*, the other leg must be determined by its sine, and may therefore have two values, — these values being the supplements of each other. Hence, this case, in general, admits of two solutions.

Thus, if ABC is the given triangle, and $BAB'C$ is a lune whose angle $B = B' = 82^\circ\ 25'$, then each of the triangles ABC, $AB'C$ will fulfil the conditions. It is evident that the two values of a (BC, $B'C$), the two of A (CAB, CAB'), and the two of c (AB, AB') are supplements of each other.

COMPUTATION

$$\begin{aligned}
\log\tan b &= 10.40646-10\\
\operatorname{colog}\tan B &= \underline{9.12428-10}\\
\log\sin a &= 9.53074-10\\
a &= 19^\circ\ 50.5'\\
\text{or}\qquad a &= 160^\circ\ 9.5'.
\end{aligned}$$

$$\begin{aligned}
\log\cos B &= 9.12047-10\\
\operatorname{colog}\cos b &= \underline{0.43753}\\
\log\sin A &= 9.55800-10\\
A &= 21^\circ\ 11.2'\\
\text{or}\qquad A &= 158^\circ\ 48.8'.
\end{aligned}$$

By the third formula, we find, in like manner,

$$c = 69^\circ\ 54.7', \text{ or } 110^\circ\ 5.3'.$$

136. Exercise XXI.

1. Given $a = 55° 28'$, $b = 63° 15'$; find $c = 75° 13'$, $A = 58° 26'$, $B = 67° 27'$.

2. Given $a = 50° 30'$, $b = 72° 50'$; find c, A, B.

3. Given $c = 110° 30'$, $a = 50° 45'$; find $b = 123° 37'$, $A = 55° 46'$, $B = 117° 14'$.

4. Given $c = 67°$, $a = 70° 18'$; find b, A, B.

5. Given $a = 151°$, $A = 125° 31'$; find $c = 36° 33'$, $b = 156° 42'$, $B = 138° 23'$, or $c' = 143° 27'$, $b' = 23° 18'$, $B' = 41° 37'$.

6. Given $b = 128° 33'$, $B = 105° 59'$; find a, c, A.

7. Given $a = 118° 54'$, $B = 12° 19'$; find $c = 118° 20'$, $b = 10° 49'$, $A = 95° 55'$.

8. Given $b = 31° 27'$, $A = 49° 51'$; find a, c, B.

9. Given $c = 69° 25.2'$, $A = 54° 54.7'$; find $a = 50°$, $b = 56° 51'$, $B = 63° 25'$.

10. Given $c = 93° 12'$, $B = 67° 34'$; find a, b, A.

11. Given $A = 113° 9.2'$, $B = 130° 18.3'$; find $a = 121° 2'$, $b = 134° 43'$, $c = 68° 44'$.

12. Given $A = 94° 25'$, $B = 59° 43'$; find a, b, c.

13. Given $c = 59° 3'$, $A = 147° 32'$; find a, b, B.

14. Given $a = 30° 32.4'$, $B = 36° 44'$; find $b = 20° 46'$, $c = 36° 22'$.

15. Given $a = 28° 47'$, $b = 110° 27.3'$; find c, A, B.

16. Given $A = 90°$, $B = 90°$; find $a = 90°$, $b = 90°$, $c = 90°$.

17. Given $a = 90°$, $b = 90°$; find $c = 90°$, $A = 90°$, $B = 90°$.

18. Given $a = 90°$, $B = 90°$; find $c = 90°$, $b = 90°$, $A = 90°$.

19. Given $a = 20°$, $A = 20°$; find $c = 90°$, $b = 90°$, $B = 90°$.

20. Given $c = 90°$, $A = 90$; find $a = 90°$, b and B indeterminate.

21. Given $c = 90°$, $a = 90°$; find $A = 90°$, b and B indeterminate.

Prove the following:

22. $\cos^2 a \cos^2 B = \sin^2 A - \sin^2 a$.

23. $\sin^2 a \sin^2 B = \sin^2 A - \cos^2 B$.

24. $\tan^2 \frac{1}{2} A = \sin(c - b) \div \sin(c + b)$.

25. If $A = 36°$ and $B = 60°$, then $a + b + c = 90°$.

137. Isosceles Triangles. By Geometry, an isosceles spherical triangle is divided into two equal right triangles by drawing the arc (y) of a great circle from the vertex to the middle point of the base (b). Therefore, an isosceles triangle is solvable when one of these right triangles can be solved.

(1) Given $a = c = 110° 47.3'$, $B = 92° 14.6'$;

solve the isosceles triangle ABC.

STATEMENT AND ANSWERS

$$\angle ABD = \tfrac{1}{2}B = 46° 7.3'.$$

$\cos c = \cot A \cot \frac{1}{2}B$, $\therefore$ $\cot A = \cos c \tan \frac{1}{2}B$; whence $A = 110° 16'$.

$\sin \frac{1}{2}b = \sin c \sin \frac{1}{2}B$; whence $b = 84° 44'$.

$\cos \frac{1}{2}B = \tan y \cot a$, $\therefore$ $\tan y = \cos \frac{1}{2}B \tan a$; whence $y = 118° 43'$.

138. Exercise XXII.

Solve the following isosceles triangles in which $a = c$:

1. Given $A = 82° 26'$, $B = 64° 42'$; find $a = 77° 54'$, $b = 63° 5'$.
2. Given $a = 112° 25'$, $b = 123° 48'$; find $A = 140° 35'$, $B = 145° 12'$.
3. Given $A = 62° 37'$, $a = 74° 13'$; find B, b.
4. Given $a = b = c$; find $\sin \frac{1}{2}A = \frac{1}{2} \sec \frac{1}{2}a$.

CHAPTER VI

OBLIQUE TRIANGLES

RELATIONS OF SIDES AND ANGLES

139. Relation of one angle to the three sides.

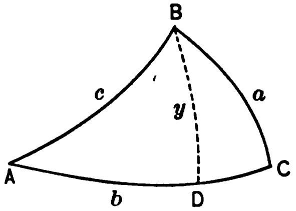

FIG. 63.

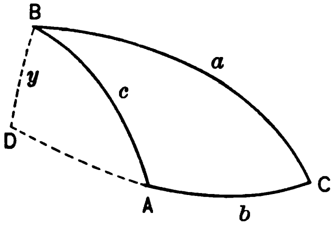

FIG. 64.

Let the angle, say A, be acute (Fig. 63), or obtuse (Fig. 64).

In each figure draw BD, the arc of a great circle, $\perp AC$, and let $y = BD$, $u = \angle ABD$.

In both figures (Art. 58)

$$AD + DC = AC, \text{ or } DC = b - AD.$$

$$\therefore \cos DC = \cos b \cos AD + \sin b \sin AD,$$

or ($\sin AD = \sin c \sin u$)

$$\cos DC = \cos b \cos AD + \sin b \sin c \sin u \quad . \quad . \quad . \quad (a)$$

$(a) \times \cos y$,

$$\cos DC \cos y = \cos b \cos AD \cos y + \sin b \sin c \sin u \cos y.$$

But $\quad \cos DC \cos y = \cos a, \cos AD \cos y = \cos c,$

and $\quad \sin u \cos y = \cos A.$

$$\therefore \boldsymbol{\cos a = \cos b \cos c + \sin b \sin c \cos A} \quad . \quad . \quad . \quad . \quad [89]$$

Cyclically,

$$\boldsymbol{\cos b = \cos c \cos a + \sin c \sin a \cos B} \quad . \quad . \quad . \quad . \quad [90]$$

$$\boldsymbol{\cos c = \cos a \cos b + \sin a \sin b \cos C} \quad . \quad . \quad . \quad . \quad [91]$$

While all other trigonometric relations of the sides and angles of spherical triangles can be derived from [89], [90], [91], it is often more convenient to deduce them by independent methods. This fundamental relation corresponds to that of Art. 96 in Plane Trigonometry, and is called the **Law of Cosines.**

140. Relation of one side to the three angles.

Applying [89] to the polar triangle $A'B'C'$ (Fig. 56),

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'.$$

But since $a' = 180° - A$, $b' = 180° - b$, $c' = 180° - C$, $A' = 180° - a$, we have, by Art. 85,

$$-\cos A = (-\cos B)(-\cos C) + \sin B \sin C(-\cos a),$$

or

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a \quad . \quad . \quad . \quad . \quad [92]$$

Cyclically,

$$\cos B = -\cos C \cos A + \sin C \sin A \cos b \quad . \quad . \quad . \quad . \quad [93]$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c \quad . \quad . \quad . \quad . \quad [94]$$

This relation, like the corresponding one in Plane Trigonometry (Art. 98), is called the **Law of Angles.**

141. Passing to Polar Triangle. The preceding process is called "applying the formula to the polar triangle" or "passing to the polar triangle." By this process any formula expressing a relation between the sides and angles of any spherical triangle may be immediately transformed into another, having angles where the original had sides, and *vice versa.*

142. Relation of two sides to their opposite angles.

In Figs. 63, 64,

(1) $$\sin y = \sin a \sin C,$$

(2) $$\sin y = \sin c \sin A.$$

$$\therefore \sin a \sin C = \sin c \sin A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [95]$$

Cyclically,

$$\sin b \sin A = \sin a \sin B, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [96]$$

$$\sin c \sin B = \sin b \sin C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [97]$$

This relation may be deduced from [89] and [90] by eliminating c, which may be done thus:

[89] + [90], $(\cos a + \cos b)(1 - \cos c) = \sin c(\sin a \cos B + \sin b \cos A)$. [98]

[89] − [90], $(\cos a - \cos b)(1 + \cos c) = \sin c(\sin a \cos B + \sin b \cos A)$. (g)

[98] × (g), $(\cos^2 a - \cos^2 b)(1 - \cos^2 c) = \sin^2 c(\sin^2 a \cos^2 B - \sin^2 b \cos^2 A)$.

Divide by $1 - \cos^2 c = \sin^2 c$, make $\cos^2 a = 1 - \sin^2 a$, $\cos^2 b = 1 - \sin^2 b$,

$$\sin^2 a - \sin^2 b = \sin^2 a \cos^2 B - \sin^2 b \cos^2 A.$$

Whence,

$$\sin a \sin B = \sin b \sin A.$$

Formula [95], [96], or [97] may be put in the form of a proportion, as

$$\sin a : \sin b :: \sin A : \sin B.$$

That is, in spherical triangles, *the sines of the sides are proportional to the sines of the opposite angles*, which property is called the **Law of Sines.**

143. Relation of the half of one angle to the three sides.

For brevity, let $2s = a + b + c$, then $a + b - c = 2(s - c)$, $a - b + c = 2(s - b)$, and $-a + b + c = 2(s - a)$.

Solving [89] for $\cos A$,

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Substituting this value of $\cos A$ in

$$(1)\ 2\sin^2 \tfrac{1}{2} A = 1 - \cos A, \text{ and } (2)\ 2\cos^2 \tfrac{1}{2} A = 1 + \cos A,$$

$$(1)\ 2\sin^2 \tfrac{1}{2} A = \frac{\sin b \sin c + \cos b \cos c - \cos a}{\sin b \sin c}$$

by [27] $$= \frac{\cos(b - c) - \cos a}{\sin b \sin c}$$

by [50] $$= \frac{2 \sin \frac{1}{2}(a + b - c) \sin \frac{1}{2}(a - b + c)}{\sin b \sin c};$$

$$(2)\ 2\cos^2 \tfrac{1}{2} A = \frac{\sin b \sin c - \cos b \cos c + \cos a}{\sin b \sin c}$$

by [23] $$= \frac{\cos a - \cos(b + c)}{\sin b \sin c}$$

in [50] put $c = -c$, $$= \frac{2 \sin \frac{1}{2}(a + b + c) \sin \frac{1}{2}(b + c - a)}{\sin b \sin c}.$$

$$\therefore\ \sin \tfrac{1}{2} A = \sqrt{\frac{\sin(s - b) \sin(s - c)}{\sin b \sin c}} \quad . \quad . \quad . \quad . \qquad [99]$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin s \sin(s - a)}{\sin b \sin c}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad [100]$$

Whence (Art. 70),

$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin(s - b) \sin(s - c)}{\sin s \sin(s - a)}} \quad . \quad . \quad . \quad . \qquad [101]$$

Cyclically, $$\sin\tfrac{1}{2}B = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin c \sin a}} \quad . \quad . \quad . \quad . \quad [102]$$

$$\cos\tfrac{1}{2}B = \sqrt{\frac{\sin s \sin(s-b)}{\sin c \sin a}} \quad . \quad . \quad . \quad . \quad . \quad . \quad [103]$$

$$\tan\tfrac{1}{2}B = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin s \sin(s-b)}} \quad . \quad . \quad . \quad . \quad [104]$$

$$\sin\tfrac{1}{2}C = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin a \sin b}} \quad . \quad . \quad . \quad . \quad [105]$$

$$\cos\tfrac{1}{2}C = \sqrt{\frac{\sin s \sin(s-c)}{\sin a \sin b}} \quad . \quad . \quad . \quad . \quad . \quad . \quad [106]$$

$$\tan\tfrac{1}{2}C = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s \sin(s-c)}} \quad . \quad . \quad . \quad . \quad [107]$$

144. Relation of the half of one side to the three angles.

These relations may be derived from [92] just as [99] to [107] were from [89].

Thus, let $2S = A + B + C$, then $A + B - C = 2(S - C)$, $A - B + C = 2(S - B)$, and $B + C - A = 2(S - A)$. Solving [92] for $\cos a$,

$$\cos a = \frac{\cos B \cos C + \cos A}{\sin B \sin C}.$$

Substitute this value of $\cos a$ for $\cos a$, in

(1) $2\sin^2\tfrac{1}{2}a = 1 - \cos a$, and (2) $2\cos^2\tfrac{1}{2}a = 1 + \cos a$, proceed in the same manner as in Art. 143, and we obtain:

$$\sin\tfrac{1}{2}a = \sqrt{-\frac{\cos S \cos(S-A)}{\sin B \sin C}} \quad . \quad . \quad . \quad . \quad . \quad [108]$$

$$\cos\tfrac{1}{2}a = \sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B \sin C}} \quad . \quad . \quad . \quad . \quad [109]$$

$$\tan\tfrac{1}{2}a = \sqrt{-\frac{\cos S \cos(S-A)}{\cos(S-B)\cos(S-C)}} \quad . \quad . \quad . \quad [110]$$

$$\sin\tfrac{1}{2}b = \sqrt{-\frac{\cos S \cos(S-B)}{\sin C \sin A}} \quad . \quad . \quad . \quad . \quad . \quad [111]$$

$$\cos\tfrac{1}{2}b = \sqrt{\frac{\cos(S-C)\cos(S-A)}{\sin C \sin A}} \quad . \quad . \quad . \quad . \quad [112]$$

$$\tan\frac{1}{2}b = \sqrt{-\frac{\cos S\cos(S-B)}{\cos(S-C)\cos(S-A)}} \quad . \quad . \quad . \quad [113]$$

$$\sin\frac{1}{2}c = \sqrt{-\frac{\cos S\cos(S-C)}{\sin A\sin B}} \quad . \quad . \quad . \quad . \quad . \quad . \quad [114]$$

$$\cos\frac{1}{2}c = \sqrt{\frac{\cos(S-A)\cos(S-B)}{\sin A\sin B}} \quad . \quad . \quad . \quad [115]$$

$$\tan\frac{1}{2}c = \sqrt{-\frac{\cos S\cos(S-C)}{\cos(S-A)\cos(S-B)}} \quad . \quad . \quad . \quad [116]$$

145. Relation of one side to the other two sides and their opposite angles.

Dividing [98] by $\sin c$ and, by [39], we have

$$(\cos a + \cos b)\tan\frac{1}{2}c = \sin a\cos B + \sin b\cos A \quad . \quad . \quad [117]$$

which is one relation of the side c to the other two sides (a, b) and their opposite angles (A, B); but let us put it under a more convenient form.

Making $$\sin a = \frac{\sin b\sin A}{\sin B},$$

$$(\cos a + \cos c)\tan\tfrac{1}{2}c = \frac{\sin b}{\sin B}(\sin A\cos B + \cos A\sin B),$$

or, by [22],

$$(\cos a + \cos b)\tan\frac{1}{2}c = \frac{\sin b}{\sin B}\sin(A+B) \quad . \quad . \quad . \quad . \quad [118]$$

This is another form of the required relation, but it may be reduced to a still more convenient form.

The proportion, $\sin a : \sin b :: \sin A : \sin B$,

taken first by composition, and then by division, gives

$$\sin a + \sin b = \frac{\sin b}{\sin B}(\sin A + \sin B) \quad . \quad . \quad . \quad . \quad (f)$$

$$\sin a - \sin b = \frac{\sin b}{\sin B}(\sin A - \sin B) \quad . \quad . \quad . \quad . \quad (g)$$

Dividing (f) by [118], and, by [46], [48], we have

$$\frac{\tan\frac{1}{2}(a+b)}{\tan\frac{1}{2}c} = \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [119]$$

Again, dividing (g) by [118], and, by [47], [49], we have

$$\frac{\tan\frac{1}{2}(a-b)}{\tan\frac{1}{2}c}=\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)} \quad . \; . \; . \; . \; . \; . \; . \quad [120]$$

Formulas [119] and [120] are known as the *first* and *second* of **Napier's Analogies.**

146. Relation of one angle to the other two angles and their opposite sides.

Applying [119] to the polar triangle $A'B'C'$, Fig. 56,

$$\frac{\tan\frac{1}{2}(a'+b')}{\tan\frac{1}{2}c'}=\frac{\cos\frac{1}{2}(A'-B')}{\cos\frac{1}{2}(A'+B')} \quad . \; . \; . \; . \; . \; . \; . \quad (f)$$

Now since $a'=180°-A$, $b'=180-B$, $c'=180°-C$, $A'=180°-a$, and $B'=180°-b$, $\frac{1}{2}(a'+b')=180°-\frac{1}{2}(A+B)$, $\frac{1}{2}c'=90°-\frac{1}{2}C$, $\frac{1}{2}(A'-B')=-\frac{1}{2}(a-b)$, and $\frac{1}{2}(A'+B')=180°-\frac{1}{2}(a+b)$. Therefore ($f$) becomes (Arts. 27, 85),

$$\frac{\tan\frac{1}{2}(A+B)}{\cot\frac{1}{2}C}=\frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)} \quad . \; . \; . \; . \; . \; . \quad [121]$$

Again, applying [120] to the polar triangle, and proceeding in a similar manner, we find

$$\frac{\tan\frac{1}{2}(A-B)}{\cot\frac{1}{2}C}=\frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)} \quad . \; . \; . \; . \; . \; . \quad [122]$$

Formulas [121] and [122] are called the *third* and *fourth* of **Napier's Analogies.**

SOLUTION OF OBLIQUE TRIANGLES

147. Solvable Triangles. It will appear as we proceed that when any three of the six parts of a spherical triangle are given, the remaining three may be found. The three given parts may be

I. *Two sides and their included angle.*
II. *Two angles and their included side.*
III. *Two sides and the angle opposite to one of them.*
IV. *Two angles and the side opposite to one of them.*
V. *The three sides.*
VI. *The three angles.*

148. How spherical triangles may be solved. If the triangle to be solved falls under Case II, IV, or VI, it may be reduced to Case I, III, or V, respectively, by passing to the polar triangle.

Thus, if the given parts are A, C, b, as in Case II, we can find a', c', B' of the polar triangle by the relations $a' = 180° - A$, $c' = 180° - C$, $B' = 180° - b$. That is, of the polar triangle the known parts are as in Case I. By solving this triangle we find A', C', b', from which we can find the unknown parts of the given triangle by the relations $a = 180° - A'$, $c = 180° - C'$, $B = 180° - b'$.

Therefore, it will be sufficient to show how Cases I, III, and V may be solved.

By using natural functions we really need only the three formulas [89], [95], and [117], or [118] or [119], and those which may be deduced from these by cyclical deduction. An example under each case must suffice to illustrate the process, as more convenient methods of solution will be subsequently presented.

Illustrative Examples. CASE I. Given $a = 98°$, $c = 60°$, $B = 110°$; find A, C, b.

STATEMENT

By [90], $\cos b = \cos a \cos c + \sin a \sin c \cos B.$

By [96], $\sin A = \sin a \sin B \div \sin b.$

By [97], $\sin C = \sin c \sin B \div \sin b.$

COMPUTATION

$$\cos b = (-.13917)(.5) + (.99027)(.86603)(-.34202) = -.06958 - .29332$$
$$= -.36290. \qquad \therefore\ b = 180° - 68° 43.3' = 111° 16.7'.$$

$$\sin A = \frac{.93969}{.93183} \times .99027 = 1.00844 \times .99027 = .998627. \qquad \therefore\ A = 86° 59.8'.$$

$$\sin C = 1.00844 \times .86603 = .873339. \qquad \therefore\ C = 60° 50.9'.$$

CASE III. Given $a = 42° 45'$, $b = 47° 15'$, $A = 56° 30'$; find B, C, c.

STATEMENT

By [96], $\sin B = \sin b \sin A \div \sin a.$

Now we may express c in terms of a, b, A, B by means of [117], [118], [119], or [120], but the last two involve the least labor.

By [120], $\tan \frac{1}{2} c = \sin \frac{1}{2}(B + A) \tan \frac{1}{2}(b - a) \div \sin \frac{1}{2}(B - A).$

By [95], $\sin C = \sin c \sin A \div \sin a.$

COMPUTATION

$$\sin B = \frac{.83389}{.67880} \times .73432 = 1.228475 \times .73432 = .902094.$$
$$\therefore\ B = 64° 26'.$$

$$\tan \tfrac{1}{2} c = \frac{.87007 \times .03929}{.06919} = .49408. \qquad \therefore\ c = 52° 35'.$$

$$\sin C = 1.228475 \times .79424 = .975704. \qquad \therefore\ C = 77° 21'.$$

CASE V. Given $a = 38°$, $b = 42°$, $c = 51°$; find A, B, C.

STATEMENT

By [89], $\cos A = (\cos a - \cos b \cos c) \div \sin b \sin c.$

By [96], $\sin B = (\sin A \div \sin a) \sin b.$

By [95], $\sin C = (\sin A \div \sin a) \sin c.$

COMPUTATION

$$\begin{aligned}\cos A &= (.78801 - .74314 \times .62932) \div .66913 \times .77715\\ &= .32034 \div .52001 = .61602. \qquad \therefore\ A = 51°\,58.4'.\\ \sin B &= \frac{.78772}{.61566} \times .66913 = 1.27947 \times .66913 = .85613.\\ &\therefore\ B = 58°\,53'.\\ \sin C &= 1.27947 \times .77715 = .99434. \qquad \therefore\ C = 83°\,54'.\end{aligned}$$

We will now consider the six cases of solvable triangles in order, and show how they may be solved by direct solutions, and by using formulas which are adapted to computation by natural functions or logarithms.

149. I. When two sides and their included angle are given.

We first find the two unknown angles by Napier's Analogies, and then find the unknown side by the law of sines or by one of Napier's Analogies.

(1) Given $a=62°\,38'$, $b=10°\,13'\,19''$, $C=150°\,24'\,12''$; find c, A, B.

STATEMENT

By [121], $\tan \frac{1}{2}(A+B) = \cos \frac{1}{2}(a-b) \cot \frac{1}{2} C \div \cos \frac{1}{2}(a+b).$

By [122], $\tan \frac{1}{2}(A-B) = \sin \frac{1}{2}(a-b) \cot \frac{1}{2} C \div \sin \frac{1}{2}(a+b).$

By [120], $\tan \frac{1}{2} c = \sin \frac{1}{2}(A+B) \tan \frac{1}{2}(a-b) \div \sin \frac{1}{2}(A-B).$

Check: $$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

COMPUTATION

$$\tfrac{1}{2}(a-b) = 26°\,12.3', \quad \tfrac{1}{2}(a+b) = 36°\,25.6', \quad \tfrac{1}{2}C = 75°\,12.1'.$$

$$\begin{aligned}\log\cos\tfrac{1}{2}(a-b) &= 9.95290-10 & \log\sin\tfrac{1}{2}(a-b) &= 9.64502-10\\ \log\cot\tfrac{1}{2}C &= 9.42190-10 & \log\cot\tfrac{1}{2}C &= 9.42190-10\\ \operatorname{colog}\cos\tfrac{1}{2}(a+b) &= \underline{0.09441} & \operatorname{colog}\sin\tfrac{1}{2}(a+b) &= \underline{0.22636}\\ \log\tan\tfrac{1}{2}(A+B) &= 9.46921-10 & \log\tan\tfrac{1}{2}(A-B) &= 9.29328-10\\ \tfrac{1}{2}(A+B) &= 16°\,24.8'. & \tfrac{1}{2}(A-B) &= 11°\,6.9'.\end{aligned}$$

$$\begin{aligned}\therefore\ A &= \tfrac{1}{2}(A+B) + \tfrac{1}{2}(A-B) = 27°\,31.7';\\ B &= \tfrac{1}{2}(A+B) - \tfrac{1}{2}(A-B) = 5°\,17.9'.\end{aligned}$$

$$\begin{aligned}\log\sin\tfrac{1}{2}(A+B) &= 9.45111\\ \log\tan\tfrac{1}{2}(a-b) &= 9.69213\\ \operatorname{colog}\sin\tfrac{1}{2}(A-B) &= \underline{0.71494} & \therefore\ \tfrac{1}{2}c &= 35°\,48.4',\\ \log\tan\tfrac{1}{2}c &= 9.85818 & c &= 71°\,37'.\end{aligned}$$

150. Exercise XXIII.

1. Given $a = 72°$, $b = 47°$, $C = 33°$; find $A = 121° 33'$, $B = 40° 57'$, $c = 37° 26'$.
2. Given $b = 78° 15'$, $c = 56° 20'$; $A = 120°$; find $B = 62° 32'$, $C = 48° 57'$, $a = 107° 8'$.
3. Given $b = 137°$, $c = 116°$, $A = 68° 33' 45''$; find $B = 130° 48'$, $C = 94° 1'$, $a = 57°$
4. Given $a = 101° 31'$, $b = 36° 16.5'$, $C = 112° 42.9'$; find $A = 78° 19'$, $B = 36° 15'$, $c = 112° 38'$.
5. Given $a = 40° 18'$, $c = 100° 42'$, $B = 62° 30'$; find A, C, b.

151. II. **When two angles and their included side are given.**

(1) Given $A = 78° 19'$, $B = 36° 15'$, $c = 112° 38'$; find a, b, C.

STATEMENT

$$\text{By [119], } \tan\tfrac{1}{2}(a+b) = \cos\tfrac{1}{2}(A-B)\tan\tfrac{1}{2}c \div \cos\tfrac{1}{2}(A+B) \quad . \quad (f)$$

$$\text{By [120], } \tan\tfrac{1}{2}(a-b) = \sin\tfrac{1}{2}(A-B)\tan\tfrac{1}{2}c \div \sin\tfrac{1}{2}(A+B) \quad . \quad (g)$$

$$\text{By [122], } \cot\tfrac{1}{2}C = \sin\tfrac{1}{2}(a+b)\tan\tfrac{1}{2}(A-B) \div \sin\tfrac{1}{2}(a-b).$$

By (f) we find the value of $\tfrac{1}{2}(a+b)$ and by (g), the value of $\tfrac{1}{2}(a-b)$; we then add these values to get a, and subtract them to get b. We may use the law of sines as a check, as in the preceding case.

COMPUTATION

$$\tfrac{1}{2}(A+B) = 57° 17', \quad \tfrac{1}{2}(A-B) = 21° 2', \quad \tfrac{1}{2}c = 56° 19'.$$

$$\begin{array}{rlrl}
\log\cos\tfrac{1}{2}(A-B) = & 9.97005-10 & \log\sin\tfrac{1}{2}(A-B) = & 9.55499-10 \\
\log\tan\tfrac{1}{2}c = & 10.17620-10 & \log\tan\tfrac{1}{2}c = & 10.17620-10 \\
\operatorname{colog}\cos\tfrac{1}{2}(A+B) = & 0.26722 & \operatorname{colog}\sin\tfrac{1}{2}(A+B) = & 0.07502 \\
\hline
\log\tan\tfrac{1}{2}(a+b) = & 10.41347-10 & \log\tan\tfrac{1}{2}(a-b) = & 9.80621-10 \\
\tfrac{1}{2}(a+b) = & 68° 53.74'. & \tfrac{1}{2}(a-b) = & 32° 37.25'.
\end{array}$$

$$\therefore\ a = 101° 31', \quad b = 36° 16.5'.$$

$$\begin{array}{rlrl}
\log\sin\tfrac{1}{2}(a+b) = & 9.96985-10 & & \\
\log\tan\tfrac{1}{2}(A-B) = & 9.58493-10 & & \\
\operatorname{colog}\sin\tfrac{1}{2}(a-b) = & 0.26835 & \therefore\ \tfrac{1}{2}C = & 56° 21.44', \\
\hline
\log\cot\tfrac{1}{2}C = & 9.82313-10 & C = & 112° 42.9'.
\end{array}$$

152. Exercise XXIV.

1. Given $A = 78°$, $B = 41°$, $c = 108°$; find $a = 95° 38'$, $b = 41° 52'$, $C = 110° 48'$.
2. Given $A = 62° 54'$, $C = 125° 20'$, $b = 56° 39.5'$; find $a = 83° 12'$, $c = 114° 30'$, $B = 48° 30'$.
3. Given $A = 101° 17' 48''$, $B = 64° 52' 50''$, $c = 28° 1' 58''$; find $a = 67° 35'$, $b = 58° 36'$, $C = 29° 54'$.
4. Given $B = 64° 26' 4''$, $C = 77° 21' 36''$, $a = 42° 45'$; find $b = 47° 15'$, $c = 53° 35' 20''$, $A = 56° 30'$.
5. Given $A = 105° 36'$, $C = 73° 50'$, $b = 120°$; find B, a, c.

153. III. **When two sides and the angle opposite to one of them are given.**

(1) Given $a = 42° 45'$, $b = 47° 15'$, $A = 56° 30'$; find B, C,

STATEMENT

$$\text{By [96]},\quad \sin B = \sin b \sin A \div \sin a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

$$\text{By [122]},\quad \tan \tfrac{1}{2} C = \sin (b - a) \cot \tfrac{1}{2} (B - A) \div \sin \tfrac{1}{2} (b + a) \quad . \quad (g)$$

$$\text{By [126]},\quad \tan \tfrac{1}{2} c = \sin \tfrac{1}{2} (B + A) \tan \tfrac{1}{2} (b - a) \div \sin \tfrac{1}{2} (B - A) \, . \quad (h)$$

Since B is to be found from its sine in (f), it will have two values if $\sin b \sin A < \sin a$, and the triangle will, in general, admit of two solutions. If $\sin b \sin A > \sin a$, there will be no solution, for then $\sin B > 1$.

In order that either of these values found for B may be admissible, it is necessary and sufficient that, when substituted in (g) and (h), they give positive values for $\tan \frac{1}{2} C$ and $\tan \frac{1}{2} c$, or, which is the same thing, that $B - A$ and $b - a$ have the same sign.

This case is analogous to Case II in Plane Trigonometry, Art. 108, though it is somewhat more complex.

COMPUTATION

$$\begin{array}{rlcrl}
\log \sin b = & 9.86588 - 10 & \qquad & \tfrac{1}{2}(b + a) = & 45°, \\
\operatorname{colog} \sin a = & 0.16826 & & \tfrac{1}{2}(b - a) = & 2° 15', \\
\log \sin A = & \underline{9.92111 - 10} & & \tfrac{1}{2}(B + A) = & 60° 28' \; 2'', \\
\log \sin B = & 9.95525 - 10 & & \tfrac{1}{2}(B - A) = & 3° 58' \; 2'', \\
\therefore B = & 64° 26' 4'', & & \tfrac{1}{2}(B' + A) = & 86° \; 1' 58'', \\
B' = & 115° 33' 56''. & & \tfrac{1}{2}(B' - A) = & 29° 30' 58''.
\end{array}$$

Since both values of B are such that $B-A$, $B'-A$, and $b-a$ are all positive, there are two solutions.

(1) When $B=64^\circ\,26'\,4''$.

$$\begin{aligned}
\log\sin\tfrac{1}{2}(b-a) &= 8.59395-10\\
\operatorname{colog}\sin\tfrac{1}{2}(b+a) &= 0.15051\\
\log\cot\tfrac{1}{2}(B-A) &= \underline{11.15894-10}\\
\log\tan\tfrac{1}{2}C &= 9.90340-10\\
\tfrac{1}{2}C &= 38^\circ\,40.8',\\
C &= 77^\circ\,21.6'.
\end{aligned}
\qquad
\begin{aligned}
\log\sin\tfrac{1}{2}(B+A) &= 9.93956-10\\
\operatorname{colog}\sin\tfrac{1}{2}(B-A) &= 1.15998\\
\log\tan\tfrac{1}{2}(b-a) &= \underline{8.59428-10}\\
\log\tan\tfrac{1}{2}c &= 9.69382-10\\
\tfrac{1}{2}c &= 26^\circ\,17.65',\\
c &= 52^\circ\,35.3'.
\end{aligned}$$

Proceeding in the same manner with the other value of B, viz., $B'=115^\circ\,33'\,56''$, we find $C'=11^\circ\,11.7'$, and $c'=9^\circ\,5.6'$.

154. Exercise XXV.

1. Given $b=99^\circ\,40'$, $c=64^\circ\,20'$, $B=95^\circ\,40'$; find $A=97^\circ\,19'$, $C=65^\circ\,29'$, $a=100^\circ\,42'$.
2. Given $a=109^\circ\,20'$, $c=82^\circ\,1'\,8''$, $A=107^\circ\,40'$; find $C=90^\circ$, $B=113^\circ\,33'$, $b=114^\circ\,48'$.
3. Given $a=99^\circ\,40'\,48''$, $b=64^\circ\,23'\,15''$, $A=95^\circ\,38'\,4''$; find $B=65^\circ\,33'$ $C=97^\circ\,26'$ $c=100^\circ\,49'$.
4. Given $a=46^\circ\,20.75'$, $b=65^\circ\,18.25'$, $A=40^\circ\,10.5'$; find $B=54^\circ\,6'$, $C=116^\circ\,55'$, $c=90^\circ\,32'$, $B'=125^\circ\,54'$, $C'=24^\circ\,13$, $c'=27^\circ\,23'$.
5. Given $a=30^\circ\,40'$, $c=67^\circ\,24'$, $A=50^\circ\,18.5'$; find C, B, b.

155. IV. When two angles and the side opposite to one of them are given.

This case reduces, by passing to the polar triangle, to the preceding case, and gives rise to the same ambiguities. Hence, the same remarks made in Art. 153 apply in this case also.

(1) Given $A=66^\circ\,7'\,20''$, $B=52^\circ\,50'\,20''$, $a=59^\circ\,28'\,27''$; find b, c, C.

STATEMENT

By [96], $\sin b=\sin B\sin a\div\sin A$ . . . . (f)

By [119], $\tan\frac{1}{2}c=\cos\frac{1}{2}(A+B)\tan\frac{1}{2}(a+b)\div\cos\frac{1}{2}(A-B)$.

By [121], $\tan\frac{1}{2}C=\cos\frac{1}{2}(a-b)\cot\frac{1}{2}(A+B)\div\cos\frac{1}{2}(a+b)$.

COMPUTATION

By (f), we find $b = 48^\circ\, 39.3'$, or $= 131^\circ\, 20.7'\ (= b')$.

The second value of b is inadmissible, since $A - B$ is positive, and $a - b'$ is negative, and hence there is only one solution.

$\frac{1}{2}(A+B) = 59^\circ\, 28.8'$,	$\frac{1}{2}(a+b) = 54^\circ\, 3.9'$,
$\frac{1}{2}(A-B) = 6^\circ\, 38.5'$,	$\frac{1}{2}(a-b) = 5^\circ\, 24.6'$.
$\log\cos\frac{1}{2}(A+B) = 9.70572-10$	$\log\cos\frac{1}{2}(a-b) = 9.99806-10$
$\log\tan\frac{1}{2}(a+b) = 10.13975-10$	$\log\cot\frac{1}{2}(A+B) = 9.77050-10$
$\text{colog}\cos\frac{1}{2}(A-B) = 0.00293$	$\text{colog}\cos\frac{1}{2}(a+b) = 0.23145$
$\log\tan\frac{1}{2}c = 9.84840-10$	$\log\tan\frac{1}{2}C = .00001$
$\frac{1}{2}c = 35^\circ\, 11.8'$,	$\frac{1}{2}C = 45^{\circ\prime}$,
$c = 70^\circ\, 23.6'$.	$C = 90^\circ$.

156. Exercise XXVI.

1. Given $B = 116^\circ$, $C = 80^\circ$, $c = 84^\circ$; find $b = 114^\circ\, 49'$, $a = 82^\circ\, 54'$, $A = 79^\circ\, 18'$.
2. Given $A = 62^\circ$, $C = 101^\circ\, 58'\, 24''$, $a = 64^\circ\, 30'$; find $c = 90^\circ$, $B = 63^\circ\, 47'$, $b = 66^\circ\, 30'$.
3. Given $B = 73^\circ$, $C = 81^\circ\, 20'$, $b = 122^\circ\, 40'$; show that the triangle is impossible.
4. Given $A = 117^\circ\, 54.4'$, $B = 45^\circ\, 8.6'$, $a = 76^\circ\, 37.5'$; find $b = 51^\circ\, 18'$, $c = 41^\circ\, 5'$, $C = 36^\circ\, 39'$.
5. Given $A = 130^\circ\, 10'$, $C = 59^\circ\, 13'$, $c = 48^\circ\, 25'$; find B, a, b.

157. V. **When the three sides are given.**

The angles are found by means of the formulas in Art. 143. If all the angles are to be computed, the formulas for the tangent are not only the most accurate in general, but the most convenient, since only four different logarithms are required. If but one angle is required, the formula for the cosine will be found to involve the least work, though it will lack precision if the angle is near 0°.

If all the angles are required, it is convenient to modify the formulas for tangents as follows:

$$\text{By [101]},\quad \tan\tfrac{1}{2}A = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)} \times \frac{\sin(s-a)}{\sin(s-a)}}$$

$$= \frac{1}{\sin(s-a)}\sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}.$$

Making $$r = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}},$$

then $$\tan\tfrac{1}{2}A = \frac{r}{\sin(s-a)}.$$

Cyclically, $$\tan\tfrac{1}{2}B = \frac{r}{\sin(s-b)}, \qquad \tan\tfrac{1}{2}C = \frac{r}{\sin(s-c)}.$$

Whence, $$\log r = \tfrac{1}{2}[\log\sin(s-a) + \log\sin(s-b) + \log\sin(s-c) + \operatorname{colog}\sin s];$$

$$\log\tan\tfrac{1}{2}A = \log r + \operatorname{colog}\sin(s-a);$$

$$\log\tan\tfrac{1}{2}B = \log r + \operatorname{colog}\sin(s-b);$$

$$\log\tan\tfrac{1}{2}C = \log r + \operatorname{colog}\sin(s-c).$$

Having found the $\log r$ by the first of these formulas, we then determine the angles in a very simple manner, as indicated by the last three formulas.

(1) Given $a = 112°\ 6'\ 42''$, $b = 127°\ 39'\ 12''$, $c = 71°\ 12'\ 42''$; find A, B, C.

COMPUTATION

$2s = 310°\ 58.6'$ $\quad s = 155°\ 29.3'$ $\quad s - a = 43°\ 22.6'$

$s - b = 27°\ 50.1'$. $\quad s - c = 84°\ 16.6'$.

$$\begin{aligned}
\log\sin(s-a) &= 9.83682 - 10\\
\log\sin(s-b) &= 9.66924 - 10\\
\log\sin(s-c) &= 9.99783 - 10\\
\operatorname{colog}\sin s &= \underline{0.38208}\\
2\log r &= 19.88597 - 20
\end{aligned}$$

$$\begin{aligned}
\log r &= 9.94298 - 10\\
\operatorname{colog}\sin(s-a) &= \underline{0.16318}\\
\log\tan\tfrac{1}{2}A &= 10.10616 - 10\\
\tfrac{1}{2}A &= 51°\ 56',\\
A &= 103°\ 52'.
\end{aligned}$$

$$\begin{aligned}
\log r &= 9.94298 - 10\\
\operatorname{colog}\sin(s-b) &= \underline{0.33076}\\
\log\tan\tfrac{1}{2}B &= 10.27374 - 10\\
\tfrac{1}{2}B &= 61°\ 58',\\
B &= 123°\ 56'.
\end{aligned}$$

$$\begin{aligned}
\log r &= 9.94298 - 10\\
\operatorname{colog}\sin(s-c) &= \underline{0.00217}\\
\log\tan\tfrac{1}{2}C &= 9.94515 - 10\\
\tfrac{1}{2}C &= 41°\ 23.5',\\
C &= 82°\ 47'.
\end{aligned}$$

158. Exercise XXVII.

1. Given $a = 38°$, $b = 42°$, $c = 51°$; find $A = 51°\ 58'$, $B = 58°\ 53'$, $C = 83°\ 55'$.
2. Given $a = 126°$, $b = 152°$, $c = 75°$; find $A = 142°\ 24'$; $B = 159°\ 16'$, $C = 133°\ 14'$.

3. Given $a = 101^\circ\,31'\,1''$, $b = 36^\circ\,16'\,29''$, $c = 112^\circ\,38'$; find $A = 78^\circ\,19'$, $B = 36^\circ\,15'$, $C = 112^\circ\,43'$.

4. Given $a = 71^\circ\,15'$, $b = 40^\circ\,35'$, $c = 39^\circ\,10'$; find $A = 130^\circ\,35'\,55''$, $B = 31^\circ\,26'\,32''$, $C = 30^\circ\,25'\,34''$.

5. Given $a = 60^\circ\,13'$, $b = 70^\circ\,24'$, $c = 80^\circ\,27'$; find A, B, C.

159. VI. **When the three angles are given.**

The sides are computed by means of formulas [108] to [116]. The formulas for the tangents are in general to be preferred, and they may be modified as in Case V.

(1) Given $A = 150^\circ$, $B = 131^\circ$, $C = 115^\circ$; find the side c.

Let us find c by the formula for cosine [115], viz.:

$$\cos\tfrac{1}{2}c = \sqrt{\frac{\cos(S-A)\cos(S-B)}{\sin A \sin B}}.$$

COMPUTATION

$$\begin{aligned} 2S &= 396^\circ, \\ S &= 198^\circ, \\ S-A &= 48, \\ S-B &= 67^\circ. \end{aligned}$$

$$\begin{aligned} \log\cos(S-A) &= 9.82551 - 10 \\ \log\cos(S-B) &= 9.59188 - 10 \\ \text{colog}\sin A &= 0.30103 \\ \text{colog}\sin B &= 0.12222 \\ &\underline{2)19.84064 - 20} \\ \log\cos\tfrac{1}{2}c &= 9.92032 - 10 \\ \tfrac{1}{2}c &= 33^\circ\,39.4', \\ c &= 67^\circ\,18.8'. \end{aligned}$$

160. Exercise XXVIII.

1. Given $A = 74^\circ$, $B = 82^\circ$, $C = 67^\circ$; find $a = 68^\circ\,46'$, $b = 73^\circ\,48'$, $c = 63^\circ\,12'$.

2. Given $A = 91^\circ\,10'$, $B = 85^\circ\,40'$, $C = 78^\circ\,30'$; find $b = 85^\circ\,49'$.

3. Given $A = 154^\circ\,4.2'$, $B = 87^\circ\,27.1'$, $C = 76^\circ\,32.4'$; find $a = 156^\circ\,12.2'$, $b = 112^\circ\,48.6'$, $c = 63^\circ\,48.9'$.

4. Given $A = 70^\circ$, $B = 132^\circ$, $C = 94^\circ\,25.89'$; find $a = 57^\circ\,54'$, $b = 137^\circ\,56.3'$, $c = 116^\circ$.

5. Given $A = 95^\circ\,37'$, $B = 113^\circ\,40'$, $C = 78^\circ\,50'$; find a, b, c.

161. Quadrantal Triangles. A spherical triangle is called quadrantal when one of its sides is equal to a quadrant. The polar triangle of a quadrantal triangle is a right spherical triangle.

Hence to solve a quadrantal triangle we may solve its polar triangle as in Chap. V, and take the supplements of the results.

AREA OF SPHERICAL TRIANGLES

162. The Spherical Excess (E) of a spherical triangle is the amount by which the sum of the three angles exceeds 180° or π. Thus,

$$A+B+C-\pi=E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

It is shown in Geometry that the areas of two spherical triangles are to each other as their spherical excesses, and also that the area of a trirectangular triangle is $\frac{1}{2}\pi r^2$, and its spherical excess 90°. Hence if K is the area of any triangle and $E°$ its spherical excess, we have

$$K : \tfrac{1}{2}\pi r^2 :: E° : 90°.$$

$$\therefore \boldsymbol{K=\left(\frac{E}{180}\right)\pi r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [123]$$

Example. On a sphere whose radius is 15 ft., a triangle has $A=78°\,3'$, $B=90°\,50.4'$, $C=100°\,21.6'$. Find K.

$$78°\,3'+90°\,50.4'+100°\,21.6'=269°\,15'=269.25°.$$

$$E=269.25-180=89.25.$$

$$K=\frac{89.25}{180}\pi(15)^2 \text{ sq. ft.} = 350.5 \text{ sq. ft.}$$

163. Lhuiller's Theorem. This elegant formula, by which the spherical excess is expressed in terms of the three sides, may be deduced as follows:

$$\begin{aligned}\tan\tfrac{1}{4}E &= \frac{\sin\frac{1}{4}(A+B+C-\pi)}{\cos\frac{1}{4}(A+B+C-\pi)}\\ &= \frac{\sin\frac{1}{2}(A+B)-\sin\frac{1}{2}(\pi-C)}{\cos\frac{1}{2}(A+B)+\cos\frac{1}{2}(\pi-C)}\\ &= \frac{\sin\frac{1}{2}(A+B)-\cos\frac{1}{2}C}{\cos\frac{1}{2}(A+B)+\sin\frac{1}{2}C}\\ &= \frac{\sin\frac{1}{2}A\cos\frac{1}{2}B+\cos\frac{1}{2}A\sin\frac{1}{2}B-\cos\frac{1}{2}C}{\cos\frac{1}{2}A\cos\frac{1}{2}B-\sin\frac{1}{2}A\sin\frac{1}{2}B+\sin\frac{1}{2}C}.\end{aligned}$$

Substituting from [99] to [107], and reducing,

$$\boldsymbol{\tan\tfrac{1}{4}E=\sqrt{\tan\tfrac{1}{2}s\,\tan\tfrac{1}{2}(s-a)\tan\tfrac{1}{2}(s-b)\tan\tfrac{1}{2}(s-c)}} \quad . \quad [124]$$

We find E by (a), Art. 162, or [124], according as the three angles or sides are given; in all other cases we solve the triangle so far as to find the angles or sides, and then apply (a) or [124]. Having found E, we obtain the area by [123].

APPLICATIONS OF SPHERICAL TRIGONOMETRY

164. The **Terrestrial Sphere** is the earth regarded as a sphere.

The distance between any two points on the surface of the terrestrial sphere is the shortest distance measured along the surface from one of the points to the other, and is therefore the arc of a great circle which joins the points. The angles between this arc and the meridians of the points determine the bearings of the points from each other.

165. Relation of the distance between two points on the surface of the terrestrial sphere to their latitudes and longitudes.

Let A and C be the points, then the distance from A to C is the arc AC of the great circle passing through the points. Let B be the pole, $EA'E'$ the equator, BAA' and BCC' meridians passing through A and C; then the arcs AA' and CC' are the latitudes of the points and the arc $A'C'$, or the angle ABC, is the difference of their longitudes. Hence, since $AB = 90° - AA'$, $CB = 90° - CC'$, and angle $ABC = A'C'$, the distance between the two points is one side (AC) of a spherical triangle (ABC), of which the opposite angle (B) is the difference of longitudes of the points, and the other two sides (AB and CB) are the complements of the latitudes of the points.

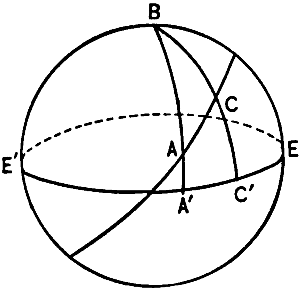

Fig. 65.

When AC has been found in degrees (say $d°$), its length in miles may be computed thus:

$$360° : d° :: \text{circumference of great circle} : \text{length } AC.$$

Taking the radius of the terrestrial sphere as 3956 miles, the circumference of a great circle is 7912π miles, and hence

$$\text{The length of } AC = \frac{d \times 7912\pi}{360} \text{ miles.}$$

166. Exercise XXIX.

1. Boston lies in lat. 42° 21′ N., lon. 71° 4′ W.; and the latitude of Greenwich is 51° 29′ N. Find the direction and the distance in miles of Greenwich from Boston. *Ans.* 3275.2 mi., N. 53° 6.5′ E.

2. New Orleans is in lat. 29° 58′, lon. 90° 3′, and Washington, D.C., is in lat. 38° 53′, lon. 77° 1′. Find the direction and distance in miles from New Orleans to Washington.

3. If a railroad were run on a straight line from San Francisco (lat. 37° 48′, lon. 122° 25′) to New York (lat. 40° 43′, lon. 74°), what would be the distance and direction?

4. Given the latitudes and longitudes of three places (*A*, *B*, *C*) on the earth's surface, and also the radius of the earth; show how to find the area of a spherical triangle formed by arcs of great circles passing through *A*, *B*, *C*.

167. The **Celestial Sphere** is an imaginary sphere of indefinite radius, upon the surface of which all the heavenly bodies appear to be situated.

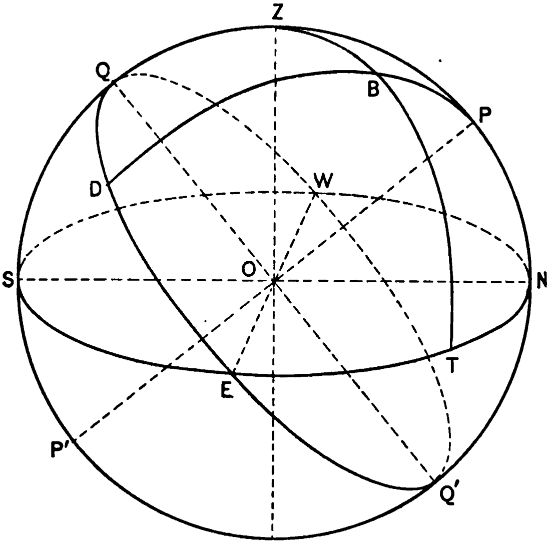

FIG. 66.

Let *O* be the position of an observer on the surface of the earth, and the sphere whose radius is *ON*, the celestial sphere.

The **Zenith** (*Z*) is the point in the surface of the celestial sphere directly over the observer's head, or the point at which a vertical or plumb line at *O* pierces the surface of the celestial sphere.

The **Poles** (*P*, *P′*) are the points in which the axis of the earth pierces the surface of the celestial sphere.

The **Horizon** is the great circle $NESW$, having Z for a pole, and the **Celestial Equator** is the great circle $EQWQ'$, whose poles are P and P'.

The **Noon-Meridian** is the vertical circle ZPN passing through P and N, N being the north point of the horizon.

Let B be a heavenly body. In consequence of the earth's rotation about its axis, B appears to describe a small circle about P as the pole, apparently moving in the direction $EQWQ'$.

Let ZBT and PBT be quadrants of great circles perpendicular, respectively, to the horizon and the celestial equator. The **Altitude** of the body B is the arc TB, the **Declination** of B is the arc DB, the **Azimuth** of B is the angle NZT, and the **Hour-Angle** of B is the angle ZPB.

The **Latitude** of the observer is the arc QZ. Since $OP = ZN$, $NP = QZ$; that is, *the elevation of the pole above the horizon is equal to the latitude of the place.*

168. Relation between the latitude of an observer and the altitude, the declination, the azimuth, and the hour-angle of a heavenly body.

In the spherical triangle ZBP, $ZB = 90° - TB = 90°$ – the altitude, $BP = 90° - DB = 90°$ – the declination, $ZP = 90° - QZ = 90°$ – the latitude, $\angle PZB =$ the azimuth, and $\angle ZPB =$ the hour-angle. That is,

The complements of the latitude, altitude, and declination are the sides of a spherical triangle of which the hour-angle and the azimuth are the angles opposite the last two sides, respectively.

If any three of these five elements are known, the other two may be determined by solving the spherical triangle.

169. To determine the hour of the day, omitting refraction.

If the altitude and declination of the sun are known, and the latitude of the observer, the three sides of the triangle BPZ are known, and the hour-angle BPZ may be computed.

If 24 hours be multiplied by the ratio of this angle to 360°, we obtain the time required for the sun to move from B to the meridian QP.

If this time be subtracted from 12 o'clock if the observation is made in the morning, or added if made in the afternoon, we obtain the *hour of the day* at the time and place of observation.

If the Greenwich time of the observation be noted on a chronometer, the difference between this and the local time as calculated above serves to determine the *longitude* of the place of observation.

In reducing time to longitude, it should be remembered that 24 hours of time correspond to 360° of longitude; that is, 1 hour of time corresponds to 15° of longitude, 1 minute to 15′, and 1 second to 15″.

170. Exercise XXX.

1. Given the latitude of a place 40° 36.4′, declination of star 23° 4.4′, its hour-angle 46° 40′; find its altitude = 47° 15′.

2. Given the declination of a star 17° 50′, its altitude 31° 18′, its azimuth 129°; find its hour-angle, and the latitude of the observer.

3. Given the latitude of a place 44° 50′ 14″, the azimuth of a star 124° 15′, and its hour-angle 20°; find its declination.

4. Given the latitude of a place 36° 34′, altitude of sun 26° 6′, its declination + 42° 42′; find the local apparent time, it being afternoon. *Ans.* 5 hr. 46 min., P.M.

5. A mariner observes the altitude of the sun to be 14° 18′, its declination being 18° 36′ N. If the latitude of the vessel is 50° 13′ N., and the observation is made in the morning, find the hour of day. If the observation is taken at 9 A.M., Greenwich time, what is the longitude of the vessel?

Ans. Time, 6 hr. 1 min.; longitude, 44° 49′ W.

FIVE-PLACE
LOGARITHMIC AND TRIGONOMETRIC
TABLES

·The MC Co·

FIVE-PLACE
LOGARITHMIC AND TRIGONOMETRIC
TABLES

WITH THE

ELEMENTARY PRINCIPLES OF LOGARITHMS

AND EXAMPLES OF THEIR USE IN
COMPUTATIONS

ARRANGED BY

J. W. NICHOLSON, A.M, LL.D.
PROFESSOR OF MATHEMATICS, LOUISIANA STATE UNIVERSITY AND
AGRICULTURAL AND MECHANICAL COLLEGE

New York
THE MACMILLAN COMPANY
LONDON: MACMILLAN & CO., Ltd.
1898

All rights reserved

COPYRIGHT, 1898,
BY THE MACMILLAN COMPANY.

Norwood Press
J. S. Cushing & Co. — Berwick & Smith
Norwood Mass. U.S.A.

PREFACE

These Tables have been prepared chiefly for use in solving the exercises in the author's text-book on Trigonometry. In compiling them the objects have been (1) to make them sufficiently accurate for educational and all ordinary practical purposes, and (2) to condense them so as to secure the great convenience of compactness. They are based on the five-figure tables of Gauss, and, while arranged in such a manner that the three tables together occupy only about forty pages, are thought to be unexcelled in clearness of typography.

For the benefit of those who have never studied the theory of logarithms, the elementary principles of logarithms, with examples of their use in numerical computations, have been inserted in the Introduction.

INTRODUCTION

PRINCIPLES OF LOGARITHMS

1. Definitions. If a and a' are any two real numbers such that

$$a = 10^{a'},$$

then a' is the logarithm of a to the base 10, which is written $a' = \log_{10} a$, or $a' = \log a$, the base being understood.

Thus, $100 = 10^2$, and $18 = 10^{1.25527\cdots}$.

$$\therefore \log 100 = 2, \text{ and } \log 18 = 1.25527\cdots.$$

That is, *the logarithm of any number, to the base* 10, *is the exponent of the power to which* 10 *must be raised to produce that number.*

Any positive number, except 1, may be taken as the base of a system of logarithms. Logarithms to the **base 10** are called **Common Logarithms,** and are the only ones used for numerical computations.

The integral part of a logarithm is called the **Characteristic,** and the decimal part the **Mantissa.**

Thus, of the logarithm of 18, 1 is the characteristic, and .25527 is the mantissa.

The **Significant Part** of a number is that part of the number which, ignoring the decimal point, embraces only and all its significant figures.

Thus, the significant part of 2400. is 24; of 2.030, 203; of .00108, 108; of .00023070, 2307.

2. Principles. I. *The logarithm of a product is equal to the sum of the logarithms of the factors.*

For, let $a = 10^{a'}$ . . (f), and $b = 10^{b'}$ . . . (g)

then $\log a = a'$, and $\log b = b'$ . . . . (h)

$(f) \times (g)$, $ab = 10^{a'+b'}$.

$\therefore$ (Art. 1), $\log(ab) = a' + b'$,

or by (h), $\log(ab) = \log a + \log b$.

3. II. *The logarithm of a quotient is equal to the logarithm of the dividend minus the logarithm of the divisor.*

For, Art. 2, $(f) \div (g)$, $\frac{a}{b} = 10^{a'-b'}$.

$\therefore$ (Art. 1), $\log\left(\frac{a}{b}\right) = a' - b'$,

or by (h), $\log\left(\frac{a}{b}\right) = \log a - \log b$.

4. III. *The logarithm of any power of a number is equal to the logarithm of the number multiplied by the exponent of the power.*

For, Art. 2, $(f)^p$, $a^p = 10^{pa'}$.

$\therefore$ (Art. 1), $\log a^p = pa'$,

or by (h), $\log a^p = p \times \log a$.

5. IV. *The logarithm of any root of a number is equal to the logarithm of the number divided by the index of the root.*

For, Art. 2, $\sqrt[r]{(f)}$, $\sqrt[r]{a} = 10^{\frac{a'}{r}}$.

$\therefore$ (Art. 1), $\log \sqrt[r]{a} = \frac{a'}{r}$,

or by (h), $\log \sqrt[r]{a} = (\log a) \div r$.

NOTE. — The preceding principles are evidently common to all logarithms, whatever may be the base; but the following belongs only to common logarithms, and this is the chief reason why they only are used in practical computations.

6. V. *The mantissas of the common logarithms of any two numbers having the same significant parts are the same.*

For, take any two numbers having the same significant parts, as 170. and .0017.

Then $170. \div .0017 = 100000 = 10^5$.

$\therefore$ (Arts. 1, 3), $\log 170. - \log .0017 = 5$.

That is, the difference between the two logarithms is an integer; hence they have different characteristics and the same mantissa.

FINDING THE COMMON LOGARITHMS OF NUMBERS

7. To find the characteristic of the common logarithm of a number.

By Algebra,

$$\begin{array}{rll}
\cdots & & \cdots \\
10^3 = 1000, & \therefore \log 1000 = 3; & \\
10^2 = 100, & \therefore \log 100 = 2; & \\
10^1 = 10, & \therefore \log 10 = 1; & \\
10^0 = 1, & \therefore \log 1 = 0 & \text{or } 10. - 10; \\
\left(\frac{1}{10} = \right) 10^{-1} = .1, & \therefore \log .1 = -1 & \text{or } 9. - 10; \\
\left(\frac{1}{10^2} = \right) 10^{-2} = .01, & \therefore \log .01 = -2 & \text{or } 8. - 10; \\
10^{-3} = .001, & \therefore \log .001 = -3 & \text{or } 7. - 10; \\
\cdots & & \cdots
\end{array}$$

Hence, the logarithm of a number between

1000 and 100 is 2 + a decimal,
100 and 10 is 1 + a decimal,
10 and 1 is 0 + a decimal.

That is, *the characteristic of the logarithm of a number greater than* 1 *is one less than the number of places to the left of the decimal point.*

Thus, $\log 325 = 2 +$ decimal, $\log 24.75 = 1 +$ decimal.

Again, the logarithm of a number between

1 and .1 is 9 + a decimal − 10,
.1 and .01 is 8 + a decimal − 10,
.01 and .001 is 7 + a decimal − 10, etc.

Hence to find the characteristic of the logarithm of a number less than 1, *subtract the number of ciphers between the decimal point and the first significant figure from* 9, *writing* − 10 *after the mantissa.*

Thus, $\log .000431 = 6 +$ the mantissa $- 10$.

8. The two integral parts of the characteristic of the logarithm of a decimal are sometimes combined and the result written before the mantissa as a negative characteristic. Thus, $7.25328 - 10 = \bar{3}.25328$, a minus sign being written over the characteristic to denote that it alone is negative, the mantissa being positive. But this form of expression is not so convenient in practice, and is becoming obsolete.

Evidently a logarithm is not changed in value by increasing either of the parts of the characteristic by 10 and decreasing the other part by 10. Thus, $7.25328-10=17.25328-20=27.25328-30$, etc.; and $3.16241=13.16241-10$, etc. Such changes are often useful in computations by logarithms. Thus, to subtract $8.17243-10$ from $3.45031-10$, we change the latter to $13.45031-20$, from this take the former, and obtain $5.27788-10$. Again, to divide $8.35416-10$ by 7, we change the logarithm to $68.35416-70$, divide this by 7, and obtain $9.76488-10$. The logarithm of a decimal is said to be in its normal form when the negative part of the characteristic is -10, and every logarithm should be reduced to this form before attempting to find the corresponding number, that is, the number of which it is the logarithm.

9. To find the logarithm of a number.

(*a*) *When the significant part of the number consists of one, two, or three figures.*

From the significant parts of the numbers in the columns headed *No.*, pages 20 to 34, find the significant part of the given number. On a line with it, and in the column having 0 at the top, is the mantissa. To this prefix the characteristic according to Art. 7. Thus:

$$\begin{array}{rlrl} \log 3 &= 0.47712, & \log .57 &= 9.75587-10, \\ \log 750 &= 2.87506, & \log .038 &= 8.57978-10, \\ \log 534 &= 2.72754, & \log 4.27 &= 0.63043, \\ \log 20500 &= 4.31175, & \log .00301 &= 7.47857-10. \end{array}$$

(*b*) *When the significant part of the number consists of four figures.*

In the column headed *No.*, pages 20 to 34, find the first three of the figures. On a line with these, and in the column having the fourth figure at the top, is the mantissa. Prefix the characteristic by Art. 7. Thus:

$$\begin{array}{rlrl} \log 3275 &= 3.51521, & \log .4035 &= 9.60584-10, \\ \log 31.04 &= 1.49192, & \log .05004 &= 8.69932-10. \end{array}$$

(*c*) *When the significant part of the number consists of more than four figures.*

Let the logarithm of 116723 be required. This number lies between 116700 and 116800, and therefore its logarithm lies between the logarithms of these numbers.

$$\begin{array}{lrcl} \text{By case } (b), & \log 116800 &=& 5.06744 \\ & \log \underline{116700} &=& \underline{5.06707} \\ \text{Differences,} & 100 && .00037 \\ \text{Let} & \log 116723 &=& 5.06707+x \\ \text{Again,} & \log \underline{116700} &=& \underline{5.06707\quad\;\;} \\ \text{Differences,} & 23 && \qquad\quad x \end{array}$$

Now by the principle of Proportional Parts* small differences in a number are proportional to the corresponding small differences in the logarithm. Hence

$$100 : 23 :: .00037 : x.$$

$$\therefore\ x = .23 \times .00037 = .00008.51 = .00009, \text{ say.}$$

$$\therefore\ \log 116723 = 5.06716.$$

This, and all similar operations, may be abbreviated thus:

$$\begin{array}{rl} \log 116700 = & 5.06707 \\ 37 \times .2\ \ = & \qquad 7.4 \\ \underline{37 \times .03} = & \underline{\qquad 1.11} \\ \log 116723 = & 5.06716 \end{array}$$

Examples.

$$\log 3423.7 = 4.53449.$$

$$\log 32.6708 = 1.51416.$$

$$\log .0015764 = 7.19767 - 10.$$

10. To find the number corresponding to a logarithm.

(*a*) *When the given mantissa is tabulated.*

Find the mantissa in the tables, pages 20 to 34. On a line with it, and in the column headed *No.*, are the first three figures of the significant part of the number, and at the top of the column containing the mantissa is the fourth figure. The decimal point must be located according to the characteristic (Art. 7).

1. Find the number whose logarithm is 2.74989.

The mantissa, page 27, corresponds to the number 5622. As the characteristic is 2, there must be 2 + 1 places to the left of the decimal point. Hence the required number is 562.2.

2. Find the number whose logarithm is 6.61204 – 10.

The mantissa, page 25, corresponds to the number 4093. As the characteristic is 6 – 10, there must be 9 – 6 or 3 ciphers between the decimal point and the first significant figure. Hence the number is .0004093.

---

* This principle is not strictly true, but it gives results that are approximately correct. For greater accuracy we must use tables of more places.

(*b*) ***When the given mantissa is not tabulated.***

1. Find the number whose logarithm is 1.58565.

The next larger tabulated mantissa is .58569, and the next smaller .58557.

$$\begin{array}{rlll} \text{The number corresponding to} & 1.58569 & \text{is} & 38.52 \\ \text{and the number corresponding to} & \underline{1.58557} & \text{is} & \underline{38.51} \\ \text{Differences,} & .00012 & & .01 \end{array}$$

$$\begin{array}{rlll} \text{Let the number corresponding to} & 1.58565 & \text{be} & 38.51 + n \\ \text{Again, the number corresponding to} & \underline{1.58557} & \text{is} & \underline{38.51\qquad} \\ \text{Differences,} & .00008 & & n \end{array}$$

By the principle of proportional parts,

$$.00012 : .00008 :: .01 : n.$$

$$\therefore\ n = \frac{.00008}{.00012} \times .01 = \frac{2}{3} \times .01 = .006.6 = .007.$$

$\therefore$ The required number is 38.517.

The work may be abbreviated thus:

Given mantissa, .58565

Next less mantissa, $\underline{.58557}$, four corresponding figures, 3851.

Tabular difference, $12\overline{)8\quad}$

$.66 = .7.$ Result, 38.517.

Examples. If $\log x = 0.92002$, $x = 8.318$.

If $\log y = 6.09167$, $y = 1235000$.

If $\log n = 7.50325 - 10$, $n = .003186$.

If $\log n = 1.48762$, $n = 30.734$.

If $\log x = 6.82326 - 10$, $x = .00066567$.

COMPUTATION BY LOGARITHMS

11. To multiply one number by another, by logarithms.

Find the value of $.0324 \times .00605 = k$, say.

Art. 2, $\log k = \log .0324 + \log .00605.$

$$\begin{aligned} \log .0324 &= 8.51055 - 10 \\ \log .00605 &= \underline{7.78176 - 10} \\ \log k &= 16.29231 - 20 = 6.29231 - 10 \\ \therefore\ k &= .00019602. \end{aligned}$$

RULE. *Find the logarithms of the factors, take their sum, and find the number whose logarithm is the sum, and it will be the required product.*

Examples. Find the values of the following by logarithms:

1. 18×37.
2. $43.8 \times .507$.
3. 5.021×825.6.
4. $.00493 \times 24.091$.
5. $.0037 \times 1.004 \times .00005209$.

Answers: 1. 666. 2. 22.207. 3. 4145.3. 4. .11877.

12. To divide one number by another, by logarithms.

Find the value of $731.25 \div .125 = k$, say.

Art. 3, $\log k = \log 731.25 - \log .125.$

$$\begin{aligned} \log 731.25 = 2.86407 &= 12.86407 - 10 \\ \log .125 &= \underline{\ \ 9.09691 - 10} \\ \log k &= \ \ 3.76716 \\ k &= 5850. \end{aligned}$$

RULE. *Find the logarithm of the dividend and also of the divisor, subtract the latter from the former, and find the number whose logarithm is the difference, and it will be the required quotient.*

Examples. Find the values of the following by logarithms:

1. $1.776 \div .24$.
2. $234.21 \div .0211$.
3. $9.25 \div .00775$.
4. $42.81 \div 34600$.
5. $(42 \times 2772) \div 924$.
6. $(13 \times 5.5654) \div 51.9$.

Answers: 1. 7.4. 2. 11100. 3. 1193.5. 4. .001237. 5. 126.

13. To raise a number to any power, by logarithms.

Find the value of $(.0624)^3 = k$, say.

Art. 4,
$$\begin{aligned} \log k &= 3 \log .0624 \\ &= 3(8.79518 - 10) \\ &= 26.38554 - 30 = 6.38554 - 10. \\ \therefore\ k &= .00024296. \end{aligned}$$

RULE. *Find the logarithm of the number, multiply it by the exponent of the power, and find the number whose logarithm is the product, and it will be the required power.*

Examples. Find the values of the following by logarithms:

1. $(3.5)^3$.
2. $(15.04)^2$.
3. $(2.5)^4$.
4. $(37.6)^3$.
5. $(.003278)^2$.
6. $(.0980937)^5$.

Answers: **1.** 42.875. **2.** 226.20. **3.** 39.062. **4.** 53159. **5.** .000010745. **6.** .0000090824.

14. To extract any root of a number, by logarithms.

Find the value of $\sqrt[5]{.075} = k$, say.

Art. 5,
$$\begin{aligned} \log k &= \tfrac{1}{5}\log .075 = \tfrac{1}{5}(8.87506 - 10) \\ &= \tfrac{1}{5}(48.87506 - 50) \\ &= 9.77501 - 10. \end{aligned}$$
$$\therefore\ k = .5956.$$

RULE. *Find the logarithm of the number, divide it by the index of the root, and find the number whose logarithm is the quotient, and it will be the required root.*

Examples. Find the values of the following by logarithms:

1. $\sqrt[2]{3}$.
2. $\sqrt[3]{17}$.
3. $\sqrt[4]{21.5}$.
4. $\sqrt[5]{.02305}$.
5. $\sqrt[7]{.00095173}$.
6. $\sqrt[2]{2^3} \times \sqrt[3]{3^2}$.
7. $\sqrt[8]{3^5} \div \sqrt[4]{4^3}$.
8. $\sqrt[10]{\frac{3}{7}}$.
9. $\sqrt[5]{\frac{2}{3}} \div \sqrt[3]{\frac{5}{3}}$.

Answers: **1.** 1.7320. **2.** 2.5713. **3.** 2.1533. **4.** .4705. **5.** .3701. **6.** 5.883. **7.** .7025. **8.** .9188. **9.** .7777.

15. The **Cologarithm** of a number is the logarithm of the reciprocal of that number.

Thus,
$$\begin{array}{rl} \operatorname{colog} 18 = \log \frac{1}{18} = & \log 1 - \log 18. \\ \log 1 \ = 0 = & 10. \qquad -10 \\ \log 18 \ = & \underline{1.25527 \qquad\quad} \\ \operatorname{colog} 18 \ = & 8.74473 - 10 \end{array}$$

Again
$$\begin{array}{rl} \operatorname{colog} .058 = & \log 1 - \log .058. \\ \log 1 \ = & 10. \qquad -10 \\ \log .058 = & \underline{8.76343 - 10} \\ \operatorname{colog} .058 = & 1.23657 \end{array}$$

It follows from the above that the cologarithm of a number may be found by subtracting its logarithm from 10.00000 − 10. This is most readily done by commencing at the left of the logarithm and subtracting each digit from 9 till we come to the last significant digit, which must be subtracted from 10, writing -10 after the result in case of a positive logarithm. In this manner the cologarithm of a number may be taken directly from the table of logarithms.

Examples. Find the following:

1. colog 23. **2.** colog 4007. **3.** colog .00517. **4.** colog .03092.

Answers: 1. 8.63827 − 10. **2.** 6.39718 − 10. **3.** 2.28651.

16. *The cologarithm of a number raised to any power is equal to the cologarithm of the number multiplied by the exponent of the power.*

For, $\operatorname{colog} n^r = \log \frac{1}{n^r} = \log \left(\frac{1}{n}\right)^r = r \log \frac{1}{n} = r \operatorname{colog} n$, which is evidently true whether r be an integer or fraction.

17. To find the value of a numerical expression involving multiplication, division, involution, and evolution.

Find the value of $\dfrac{5 \times \sqrt[3]{7} \times 6^{\frac{2}{11}}}{4 \times 13^{\frac{8}{9}}} = k$, say.

$$k = 5 \times \sqrt[3]{7} \times 6^{\frac{2}{11}} \times \frac{1}{4} \times \frac{1}{13^{\frac{8}{9}}}.$$

$$\therefore\ \log k = \log 5 + \tfrac{1}{3} \log 7 + \tfrac{2}{11} \log 6 + \operatorname{colog} 4 + \tfrac{8}{9} \operatorname{colog} 13.$$

$$\therefore\ \log k = 9.52992 - 10. \qquad \therefore\ k = .33878.$$

Rule. *Add together the logarithms of the factors of the numerator or dividend and the cologarithms of the factors of the denominator or divisor, and find the number of which the sum is the logarithm, and it will be the value required.*

Examples. Find the values of the following by logarithms:

1. $\dfrac{15.008 \times .0843}{.06376 \times 6.248}$.

2. $\dfrac{2563 \times .03442}{714.8 \times .511}$.

3. $\dfrac{\sqrt[2]{5.955} \times \sqrt[3]{61.2}}{\sqrt[5]{298.54}}$.

4. $\dfrac{\sqrt[3]{.008193} \times (.06285)^{\frac{3}{2}}}{.98342}$.

5. $\dfrac{0.007 \times 57.83 \times 2.813}{9.317 \times 00.28 \times 476.5}$.

6. $\left[\sqrt[6]{\dfrac{7}{8}} \times \sqrt[5]{\dfrac{6}{7}}\right] \div \sqrt[4]{\dfrac{5}{6}}$.

Answers: 1. 3.176. **2.** .2415. **3.** 3.076. **4.** .00323.

LOGARITHMIC FUNCTIONS

18. Table II. By this table, pp. 36 to 58, we can find approximately the logarithmic sin, cos, tan, or cot of any acute angle.

In explaining how to use the table, the two divisions into which each page is divided will be called *Sections*.

19. To find the logarithmic sin, cos, tan, or cot of an acute angle.

(*a*) *When the angle is tabulated.*

If the angle is less than 45°, the degrees will be found at the top of the page, the minutes in the column on the left of each section, and the functions, on a line with the minutes, in the column designated by the name at the top. If the angle is greater than 45°, the degrees will be found at the bottom of the page, the minutes in the column on the right of each section, and the function, on a line with the minutes, in the column designated by the name at the bottom.

In general, the characteristic is the same throughout each column, and is printed at the top and bottom of the column. When it is not the same, as for sin, tan, cot, p. 36, the change is marked by a **star**, and the characteristic is to be taken from the top or bottom according as the logarithm is above or below the bottom of the **star**.

To avoid negative characteristics, the characteristic of every logarithm is printed 10 too large, hence -10 is to be written after each logarithm. Thus:

$\log \sin 30° 10' = 9.70115 - 10$ $\log \cos 3° 26' = 9.99922 - 10$

$\log \tan 50' = 8.16273 - 10$ $\log \cos 70° 20' = 9.52705 - 10$

(*b*) *When the angle is not tabulated.*

Find the $\log \sin 20° 25' 12''$. $[25' 12'' = 25.2']$

The logarithm lies between $\log \sin 20° 25'$ and $\log \sin 20° 26'$.

The difference between these two logarithms, page 46, is .00034, which corresponds to a difference of 1′ in the angle. Hence by the principle of proportional parts,* the difference in the logarithm corresponding to a difference of .2′ is

$$.2 \times .00034 = .000068 = .00007, \text{ say.}$$

$$\therefore \log \sin 20° 25' 12'' = \log \sin 20° 25' + .00007 = 9.54270 - 10.$$

---

* The principle that small differences in angles are proportional to the corresponding small differences in their logarithms, is not strictly true, but is suffi-

The work may be abbreviated as in the next example. The difference between two consecutive tabulated functions, like 34, is called the Tabular Difference (tab. dif.).

Find $\log\cos 10^\circ 13' 25''$. $[13'25'' = 13.4']$

$$\begin{aligned} \log\cos 10^\circ 13' &= 9.99306 - 10, \text{ tab. dif. } 2.\\ .4 \times 2 = .8 &= \underline{\quad\quad 1 \quad\quad}\\ \therefore \log\cos 10^\circ 13' 25'' &= 9.99305 - 10 \end{aligned}$$

NOTE. — In this case we subtract the difference, since the cos of an acute angle decreases as the angle increases. So, too, we subtract in finding log cot, but add in finding log sin or log tan.

Find $\log\tan 86^\circ 19' 35''$. $[19'35'' = 19.6']$

$$\begin{aligned} \log\tan 86^\circ 19' &= 11.19128 - 10, \text{ tab. dif. } 198.\\ .6 \times 198 &= \underline{\quad\quad 119 \quad\quad}\\ \therefore \log\tan 86^\circ 19' 35'' &= 11.19247 - 10 \end{aligned}$$

20. To find an acute angle when its logarithmic sin, cos, tan, or cot is given.

Find the angle whose $\log\sin = 9.45359 - 10$.

The operation consists in reversing the process of Art. 19, and may be abbreviated thus:

The given mantissa, 45359

The next smaller mantissa, $\underline{45334}$, tab. dif. 43, angle 16° 30′.

25

$$\frac{25}{43} \times 1' = .6' \text{ (or more nearly } 35''). \quad \therefore \text{ Angle} = 16^\circ 30.6'.$$

Find the angle whose $\log\cot = 10.73478 - 10$.

The given mantissa, 73478

The next smaller mantissa, $\underline{73415}$, tab. dif. 71, angle 10° 27′.

63

$$\frac{63}{71} \times 1' = .9'. \quad \therefore \text{ The angle} = 10^\circ 27' - .9' = 10^\circ 26.1'.$$

NOTE. — In finding the angle corresponding to log sin or log tan, we add the difference, but subtract for log cos or log cot.

---

ciently exact for practical purposes, unless the angle is very near 0° or 90°. For closer work, tables of more places and at smaller intervals, such as those of Vega, should be used.

NATURAL FUNCTIONS

21. Table. By this table, pages 59 to 61, we can find approximately the natural sin, cos, tan, or cot of any acute angle.

Its use is similar to that of the table of logarithmic functions.

1. Find $\tan 41^\circ 27' 14''$, or $\tan 41^\circ 27.2\frac{1}{3}'$.

$$\tan 41^\circ 20' = .87955.$$

$$\tan 41^\circ 30' - \tan 41^\circ 20' = 518.$$

Correction for $7.2\frac{1}{3}'$, $51.8 \times 7.2\frac{1}{3} = 375$, nearly.

$$\underline{.87955}$$

Required value, $.88330$

2. Find the angle whose cos is .45854.

$$\cos = 45854$$

$$\text{Next less } \cos = \underline{45658}, \text{ angle } 62^\circ 50'.$$

$$196$$

$$\cos 62^\circ 40' - \cos 62^\circ 50' = 259.$$

Correction in minutes, 25.9)196(7.6′, nearly.

Required angle, $62^\circ 50' - 7.6' = 62^\circ 42.4'$.

TABLE I.

THE

COMMON OR BRIGGS LOGARITHMS

OF THE

NATURAL NUMBERS

From 1 to 10000.

No.	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	10 346
127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	12 352
133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	14 922	14 953	14 983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
144	15 836	15 866	15 897	15 927	15 957	15 987	16 017	16 047	16 077	16 107
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	17 319	17 348	17 377	17 406	17 435	17 464	17 493	17 522	17 551	17 580
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
151	17 898	17 926	17 955	17 984	18 013	18 041	18 070	18 099	18 127	18 156
152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385

No.	0	1	2	3	4	5	6	7	8	9
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
162	20 952	20 978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
170	23 045	23 070	23 096	23 121	23 147	23 172	23 198	23 223	23 249	23 274
171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
172	23 553	23 578	23 603	23 629	23 654	23 679	23 704	23 729	23 754	23 779
173	23 805	23 830	23 855	23 880	23 905	23 930	23 955	23 980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	24 254	24 279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	24 551	24 576	24 601	24 625	24 650	24 674	24 699	24 724	24 748	24 773
177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	25 261
179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	25 768	25 792	25 816	25 840	25 864	25 888	25 912	25 935	25 959	25 983
182	26 007	26 031	26 055	26 079	26 102	26 126	26 150	26 174	26 198	26 221
183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	26 951	26 975	26 998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
201	30 320	30 341	30 363	30 384	30 406	30 428	30 449	30 471	30 492	30 514
202	30 535	30 557	30 578	30 600	30 621	30 643	30 664	30 685	30 707	30 728
203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
213	32 838	32 858	32 879	32 899	32 919	32 940	32 960	32 980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
217	33 646	33 666	33 686	33 706	33 726	33 746	33 766	33 786	33 806	33 826
218	33 846	33 866	33 885	33 905	33 925	33 945	33 965	33 985	34 005	34 025
219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	34 203	34 223

No.	0	1	2	3	4	5	6	7	8	9
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
228	35 793	35 813	35 832	35 851	35 870	35 889	35 908	35 927	35 946	35 965
229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
239	37 840	37 858	37 876	37 894	37 912	37 931	37 949	37 967	37 985	38 003
240	38 021	38 039	38 057	38 075	38 093	38 112	38 130	38 148	38 166	38 184
241	38 202	38 220	38 238	38 256	38 274	38 292	38 310	38 328	38 346	38 364
242	38 382	38 399	38 417	38 435	38 453	38 471	38 489	38 507	38 525	38 543
243	38 561	38 578	38 596	38 614	38 632	38 650	38 668	38 686	38 703	38 721
244	38 739	38 757	38 775	38 792	38 810	38 828	38 846	38 863	38 881	38 899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
251	39 967	39 985	40 002	40 019	40 037	40 054	40 071	40 088	40 106	40 123
252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
255	40 654	40 671	40 688	40 705	40 722	40 739	40 756	40 773	40 790	40 807
256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
257	40 993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
259	41 330	41 347	41 363	41 380	41 397	41 414	41 430	41 447	41 464	41 481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
261	41 664	41 681	41 697	41 714	41 731	41 747	41 764	41 780	41 797	41 814
262	41 830	41 847	41 863	41 880	41 896	41 913	41 929	41 946	41 963	41 979
263	41 996	42 012	42 029	42 045	42 062	42 078	42 095	42 111	42 127	42 144
264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
268	42 813	42 830	42 846	42 862	42 878	42 894	42 911	42 927	42 943	42 959
269	42 975	42 991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
270	43 136	43 152	43 169	43 185	43 201	43 217	43 233	43 249	43 265	43 281
271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
272	43 457	43 473	43 489	43 505	43 521	43 537	43 553	43 569	43 584	43 600
273	43 616	43 632	43 648	43 664	43 680	43 696	43 712	43 727	43 743	43 759
274	43 775	43 791	43 807	43 823	43 838	43 854	43 870	43 886	43 902	43 917
275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	44 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
279	44 560	44 576	44 592	44 607	44 623	44 638	44 654	44 669	44 685	44 700

No.	0	1	2	3	4	5	6	7	8	9
280	44 716	44 731	44 747	44 762	44 778	44 793	44 809	44 824	44 840	44 855
281	44 871	44 886	44 902	44 917	44 932	44 948	44 963	44 979	44 994	45 010
282	45 025	45 040	45 056	45 071	45 086	45 102	45 117	45 133	45 148	45 163
283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
284	45 332	45 347	45 362	45 378	45 393	45 408	45 423	45 439	45 454	45 469
285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	45 637	45 652	45 667	45 682	45 697	45 712	45 728	45 743	45 758	45 773
287	45 788	45 803	45 818	45 834	45 849	45 864	45 879	45 894	45 909	45 924
288	45 939	45 954	45 969	45 984	46 000	46 015	46 030	46 045	46 060	46 075
289	46 090	46 105	46 120	46 135	46 150	46 165	46 180	46 195	46 210	46 225
290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
294	46 835	46 850	46 864	46 879	46 894	46 909	46 923	46 938	46 953	46 967
295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	47 159	47 173	47 188	47 202	47 217	47 232	47 246	47 261
297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
299	47 567	47 582	47 596	47 611	47 625	47 640	47 654	47 669	47 683	47 698
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
301	47 857	47 871	47 885	47 900	47 914	47 929	47 943	47 958	47 972	47 986
302	48 001	48 015	48 029	48 044	48 058	48 073	48 087	48 101	48 116	48 130
303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
304	48 287	48 302	48 316	48 330	48 344	48 359	48 373	48 387	48 401	48 416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	48 572	48 586	48 601	48 615	48 629	48 643	48 657	48 671	48 686	48 700
307	48 714	48 728	48 742	48 756	48 770	48 785	48 799	48 813	48 827	48 841
308	48 855	48 869	48 883	48 897	48 911	48 926	48 940	48 954	48 968	48 982
309	48 996	49 010	49 024	49 038	49 052	49 066	49 080	49 094	49 108	49 122
310	49 136	49 150	49 164	49 178	49 192	49 206	49 220	49 234	49 248	49 262
311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
312	49 415	49 429	49 443	49 457	49 471	49 485	49 499	49 513	49 527	49 541
313	49 554	49 568	49 582	49 596	49 610	49 624	49 638	49 651	49 665	49 679
314	49 693	49 707	49 721	49 734	49 748	49 762	49 776	49 790	49 803	49 817
315	49 831	49 845	49 859	49 872	49 886	49 900	49 914	49 927	49 941	49 955
316	49 969	49 982	49 996	50 010	50 024	50 037	50 051	50 065	50 079	50 092
317	50 106	50 120	50 133	50 147	50 161	50 174	50 188	50 202	50 215	50 229
318	50 243	50 256	50 270	50 284	50 297	50 311	50 325	50 338	50 352	50 365
319	50 379	50 393	50 406	50 420	50 433	50 447	50 461	50 474	50 488	50 501
320	50 515	50 529	50 542	50 556	50 569	50 583	50 596	50 610	50 623	50 637
321	50 651	50 664	50 678	50 691	50 705	50 718	50 732	50 745	50 759	50 772
322	50 786	50 799	50 813	50 826	50 840	50 853	50 866	50 880	50 893	50 907
323	50 920	50 934	50 947	50 961	50 974	50 987	51 001	51 014	51 028	51 041
324	51 055	51 068	51 081	51 095	51 108	51 121	51 135	51 148	51 162	51 175
325	51 188	51 202	51 215	51 228	51 242	51 255	51 268	51 282	51 295	51 308
326	51 322	51 335	51 348	51 362	51 375	51 388	51 402	51 415	51 428	51 441
327	51 455	51 468	51 481	51 495	51 508	51 521	51 534	51 548	51 561	51 574
328	51 587	51 601	51 614	51 627	51 640	51 654	51 667	51 680	51 693	51 706
329	51 720	51 733	51 746	51 759	51 772	51 786	51 799	51 812	51 825	51 838
330	51 851	51 865	51 878	51 891	51 904	51 917	51 930	51 943	51 957	51 970
331	51 983	51 996	52 009	52 022	52 035	52 048	52 061	52 075	52 088	52 101
332	52 114	52 127	52 140	52 153	52 166	52 179	52 192	52 205	52 218	52 231
333	52 244	52 257	52 270	52 284	52 297	52 310	52 323	52 336	52 349	52 362
334	52 375	52 388	52 401	52 414	52 427	52 440	52 453	52 466	52 479	52 492
335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
336	52 634	52 647	52 660	52 673	52 686	52 699	52 711	52 724	52 737	52 750
337	52 763	52 776	52 789	52 802	52 815	52 827	52 840	52 853	52 866	52 879
338	52 892	52 905	52 917	52 930	52 943	52 956	52 969	52 982	52 994	53 007
339	53 020	53 033	53 046	53 058	53 071	53 084	53 097	53 110	53 122	53 135

No.	0	1	2	3	4	5	6	7	8	9
340	53 148	53 161	53 173	53 186	53 199	53 212	53 224	53 237	53 250	53 263
341	53 275	53 288	53 301	53 314	53 326	53 339	53 352	53 364	53 377	53 390
342	53 403	53 415	53 428	53 441	53 453	53 466	53 479	53 491	53 504	53 517
343	53 529	53 542	53 555	53 567	53 580	53 593	53 605	53 618	53 631	53 643
344	53 656	53 668	53 681	53 694	53 706	53 719	53 732	53 744	53 757	53 769
345	53 782	53 794	53 807	53 820	53 832	53 845	53 857	53 870	53 882	53 895
346	53 908	53 920	53 933	53 945	53 958	53 970	53 983	53 995	54 008	54 020
347	54 033	54 045	54 058	54 070	54 083	54 095	54 108	54 120	54 133	54 145
348	54 158	54 170	54 183	54 195	54 208	54 220	54 233	54 245	54 258	54 270
349	54 283	54 295	54 307	54 320	54 332	54 345	54 357	54 370	54 382	54 394
350	54 407	54 419	54 432	54 444	54 456	54 469	54 481	54 494	54 506	54 518
351	54 531	54 543	54 555	54 568	54 580	54 593	54 605	54 617	54 630	54 642
352	54 654	54 667	54 679	54 691	54 704	54 716	54 728	54 741	54 753	54 765
353	54 777	54 790	54 802	54 814	54 827	54 839	54 851	54 864	54 876	54 888
354	54 900	54 913	54 925	54 937	54 949	54 962	54 974	54 986	54 998	55 011
355	55 023	55 035	55 047	55 060	55 072	55 084	55 096	55 108	55 121	55 133
356	55 145	55 157	55 169	55 182	55 194	55 206	55 218	55 230	55 242	55 255
357	55 267	55 279	55 291	55 303	55 315	55 328	55 340	55 352	55 364	55 376
358	55 388	55 400	55 413	55 425	55 437	55 449	55 461	55 473	55 485	55 497
359	55 509	55 522	55 534	55 546	55 558	55 570	55 582	55 594	55 606	55 618
360	55 630	55 642	55 654	55 666	55 678	55 691	55 703	55 715	55 727	55 739
361	55 751	55 763	55 775	55 787	55 799	55 811	55 823	55 835	55 847	55 859
362	55 871	55 883	55 895	55 907	55 919	55 931	55 943	55 955	55 967	55 979
363	55 991	56 003	56 015	56 027	56 038	56 050	56 062	56 074	56 086	56 098
364	56 110	56 122	56 134	56 146	56 158	56 170	56 182	56 194	56 205	56 217
365	56 229	56 241	56 253	56 265	56 277	56 289	56 301	56 312	56 324	56 336
366	56 348	56 360	56 372	56 384	56 396	56 407	56 419	56 431	56 443	56 455
367	56 467	56 478	56 490	56 502	56 514	56 526	56 538	56 549	56 561	56 573
368	56 585	56 597	56 608	56 620	56 632	56 644	56 656	56 667	56 679	56 691
369	56 703	56 714	56 726	56 738	56 750	56 761	56 773	56 785	56 797	56 808
370	56 820	56 832	56 844	56 855	56 867	56 879	56 891	56 902	56 914	56 926
371	56 937	56 949	56 961	56 972	56 984	56 996	57 008	57 019	57 031	57 043
372	57 054	57 066	57 078	57 089	57 101	57 113	57 124	57 136	57 148	57 159
373	57 171	57 183	57 194	57 206	57 217	57 229	57 241	57 252	57 264	57 276
374	57 287	57 299	57 310	57 322	57 334	57 345	57 357	57 368	57 380	57 392
375	57 403	57 415	57 426	57 438	57 449	57 461	57 473	57 484	57 496	57 507
376	57 519	57 530	57 542	57 553	57 565	57 576	57 588	57 600	57 611	57 623
377	57 634	57 646	57 657	57 669	57 680	57 692	57 703	57 715	57 726	57 738
378	57 749	57 761	57 772	57 784	57 795	57 807	57 818	57 830	57 841	57 852
379	57 864	57 875	57 887	57 898	57 910	57 921	57 933	57 944	57 955	57 967
380	57 978	57 990	58 001	58 013	58 024	58 035	58 047	58 058	58 070	58 081
381	58 092	58 104	58 115	58 127	58 138	58 149	58 161	58 172	58 184	58 195
382	58 206	58 218	58 229	58 240	58 252	58 263	58 274	58 286	58 297	58 309
383	58 320	58 331	58 343	58 354	58 365	58 377	58 388	58 399	58 410	58 422
384	58 433	58 444	58 456	58 467	58 478	58 490	58 501	58 512	58 524	58 535
385	58 546	58 557	58 569	58 580	58 591	58 602	58 614	58 625	58 636	58 647
386	58 659	58 670	58 681	58 692	58 704	58 715	58 726	58 737	58 749	58 760
387	58 771	58 782	58 794	58 805	58 816	58 827	58 838	58 850	58 861	58 872
388	58 883	58 894	58 906	58 917	58 928	58 939	58 950	58 961	58 973	58 984
389	58 995	59 006	59 017	59 028	59 040	59 051	59 062	59 073	59 084	59 095
390	59 106	59 118	59 129	59 140	59 151	59 162	59 173	59 184	59 195	59 207
391	59 218	59 229	59 240	59 251	59 262	59 273	59 284	59 295	59 306	59 318
392	59 329	59 340	59 351	59 362	59 373	59 384	59 395	59 406	59 417	59 428
393	59 439	59 450	59 461	59 472	59 483	59 494	59 506	59 517	59 528	59 539
394	59 550	59 561	59 572	59 583	59 594	59 605	59 616	59 627	59 638	59 649
395	59 660	59 671	59 682	59 693	59 704	59 715	59 726	59 737	59 748	59 759
396	59 770	59 780	59 791	59 802	59 813	59 824	59 835	59 846	59 857	59 868
397	59 879	59 890	59 901	59 912	59 923	59 934	59 945	59 956	59 966	59 977
398	59 988	59 999	60 010	60 021	60 032	60 043	60 054	60 065	60 076	60 086
399	60 097	60 108	60 119	60 130	60 141	60 152	60 163	60 173	60 184	60 195

No.	0	1	2	3	4	5	6	7	8	9
400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	60 304
401	60 314	60 325	60 336	60 347	60 358	60 369	60 379	60 390	60 401	60 412
402	60 423	60 433	60 444	60 455	60 466	60 477	60 487	60 498	60 509	60 520
403	60 531	60 541	60 552	60 563	60 574	60 584	60 595	60 606	60 617	60 627
404	60 638	60 649	60 660	60 670	60 681	60 692	60 703	60 713	60 724	60 735
405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
406	60 853	60 863	60 874	60 885	60 895	60 906	60 917	60 927	60 938	60 949
407	60 959	60 970	60 981	60 991	61 002	61 013	61 023	61 034	61 045	61 055
408	61 066	61 077	61 087	61 098	61 109	61 119	61 130	61 140	61 151	61 162
409	61 172	61 183	61 194	61 204	61 215	61 225	61 236	61 247	61 257	61 268
410	61 278	61 289	61 300	61 310	61 321	61 331	61 342	61 352	61 363	61 374
411	61 384	61 395	61 405	61 416	61 426	61 437	61 448	61 458	61 469	61 479
412	61 490	61 500	61 511	61 521	61 532	61 542	61 553	61 563	61 574	61 584
413	61 595	61 606	61 616	61 627	61 637	61 648	61 658	61 669	61 679	61 690
414	61 700	61 711	61 721	61 731	61 742	61 752	61 763	61 773	61 784	61 794
415	61 805	61 815	61 826	61 836	61 847	61 857	61 868	61 878	61 888	61 899
416	61 909	61 920	61 930	61 941	61 951	61 962	61 972	61 982	61 993	62 003
417	62 014	62 024	62 034	62 045	62 055	62 066	62 076	62 086	62 097	62 107
418	62 118	62 128	62 138	62 149	62 159	62 170	62 180	62 190	62 201	62 211
419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 315
420	62 325	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
421	62 428	62 439	62 449	62 459	62 469	62 480	62 490	62 500	62 511	62 521
422	62 531	62 542	62 552	62 562	62 572	62 583	62 593	62 603	62 613	62 624
423	62 634	62 644	62 655	62 665	62 675	62 685	62 696	62 706	62 716	62 726
424	62 737	62 747	62 757	62 767	62 778	62 788	62 798	62 808	62 818	62 829
425	62 839	62 849	62 859	62 870	62 880	62 890	62 900	62 910	62 921	62 931
426	62 941	62 951	62 961	62 972	62 982	62 992	63 002	63 012	63 022	63 033
427	63 043	63 053	63 063	63 073	63 083	63 094	63 104	63 114	63 124	63 134
428	63 144	63 155	63 165	63 175	63 185	63 195	63 205	63 215	63 225	63 236
429	63 246	63 256	63 266	63 276	63 286	63 296	63 306	63 317	63 327	63 337
430	63 347	63 357	63 367	63 377	63 387	63 397	63 407	63 417	63 428	63 438
431	63 448	63 458	63 468	63 478	63 488	63 498	63 508	63 518	63 528	63 538
432	63 548	63 558	63 568	63 579	63 589	63 599	63 609	63 619	63 629	63 639
433	63 649	63 659	63 669	63 679	63 689	63 699	63 709	63 719	63 729	63 739
434	63 749	63 759	63 769	63 779	63 789	63 799	63 809	63 819	63 829	63 839
435	63 849	63 859	63 869	63 879	63 889	63 899	63 909	63 919	63 929	63 939
436	63 949	63 959	63 969	63 979	63 988	63 998	64 008	64 018	64 028	64 038
437	64 048	64 058	64 068	64 078	64 088	64 098	64 108	64 118	64 128	64 137
438	64 147	64 157	64 167	64 177	64 187	64 197	64 207	64 217	64 227	64 237
439	64 246	64 256	64 266	64 276	64 286	64 296	64 306	64 316	64 326	64 335
440	64 345	64 355	64 365	64 375	64 385	64 395	64 404	64 414	64 424	64 434
441	64 444	64 454	64 464	64 473	64 483	64 493	64 503	64 513	64 523	64 532
442	64 542	64 552	64 562	64 572	64 582	64 591	64 601	64 611	64 621	64 631
443	64 640	64 650	64 660	64 670	64 680	64 689	64 699	64 709	64 719	64 729
444	64 738	64 748	64 758	64 768	64 777	64 787	64 797	64 807	64 816	64 826
445	64 836	64 846	64 856	64 865	64 875	64 885	64 895	64 904	64 914	64 924
446	64 933	64 943	64 953	64 963	64 972	64 982	64 992	65 002	65 011	65 021
447	65 031	65 040	65 050	65 060	65 070	65 079	65 089	65 099	65 108	65 118
448	65 128	65 137	65 147	65 157	65 167	65 176	65 186	65 196	65 205	65 215
449	65 225	65 234	65 244	65 254	65 263	65 273	65 283	65 292	65 302	65 312
450	65 321	65 331	65 341	65 350	65 360	65 369	65 379	65 389	65 398	65 408
451	65 418	65 427	65 437	65 447	65 456	65 466	65 475	65 485	65 495	65 504
452	65 514	65 523	65 533	65 543	65 552	65 562	65 571	65 581	65 591	65 600
453	65 610	65 619	65 629	65 639	65 648	65 658	65 667	65 677	65 686	65 696
454	65 706	65 715	65 725	65 734	65 744	65 753	65 763	65 772	65 782	65 792
455	65 801	65 811	65 820	65 830	65 839	65 849	65 858	65 868	65 877	65 887
456	65 896	65 906	65 916	65 925	65 935	65 944	65 954	65 963	65 973	65 982
457	65 992	66 001	66 011	66 020	66 030	66 039	66 049	66 058	66 068	66 077
458	66 087	66 096	66 106	66 115	66 124	66 134	66 143	66 153	66 162	66 172
459	66 181	66 191	66 200	66 210	66 219	66 229	66 238	66 247	66 257	66 266

No.	0	1	2	3	4	5	6	7	8	9
460	66 276	66 285	66 295	66 304	66 314	66 323	66 332	66 342	66 351	66 361
461	66 370	66 380	66 389	66 398	66 408	66 417	66 427	66 436	66 445	66 455
462	66 464	66 474	66 483	66 492	66 502	66 511	66 521	66 530	66 539	66 549
463	66 558	66 567	66 577	66 586	66 596	66 605	66 614	66 624	66 633	66 642
464	66 652	66 661	66 671	66 680	66 689	66 699	66 708	66 717	66 727	66 736
465	66 745	66 755	66 764	66 773	66 783	66 792	66 801	66 811	66 820	66 829
466	66 839	66 848	66 857	66 867	66 876	66 885	66 894	66 904	66 913	66 922
467	66 932	66 941	66 950	66 960	66 969	66 978	66 987	66 997	67 006	67 015
468	67 025	67 034	67 043	67 052	67 062	67 071	67 080	67 089	67 099	67 108
469	67 117	67 127	67 136	67 145	67 154	67 164	67 173	67 182	67 191	67 201
470	67 210	67 219	67 228	67 237	67 247	67 256	67 265	67 274	67 284	67 293
471	67 302	67 311	67 321	67 330	67 339	67 348	67 357	67 367	67 376	67 385
472	67 394	67 403	67 413	67 422	67 431	67 440	67 449	67 459	67 468	67 477
473	67 486	67 495	67 504	67 514	67 523	67 532	67 541	67 550	67 560	67 569
474	67 578	67 587	67 596	67 605	67 614	67 624	67 633	67 642	67 651	67 660
475	67 669	67 679	67 688	67 697	67 706	67 715	67 724	67 733	67 742	67 752
476	67 761	67 770	67 779	67 788	67 797	67 806	67 815	67 825	67 834	67 843
477	67 852	67 861	67 870	67 879	67 888	67 897	67 906	67 916	67 925	67 934
478	67 943	67 952	67 961	67 970	67 979	67 988	67 997	68 006	68 015	68 024
479	68 034	68 043	68 052	68 061	68 070	68 079	68 088	68 097	68 106	68 115
480	68 124	68 133	68 142	68 151	68 160	68 169	68 178	68 187	68 196	68 205
481	68 215	68 224	68 233	68 242	68 251	68 260	68 269	68 278	68 287	68 296
482	68 305	68 314	68 323	68 332	68 341	68 350	68 359	68 368	68 377	68 386
483	68 395	68 404	68 413	68 422	68 431	68 440	68 449	68 458	68 467	68 476
484	68 485	68 494	68 502	68 511	68 520	68 529	68 538	68 547	68 556	68 565
485	68 574	68 583	68 592	68 601	68 610	68 619	68 628	68 637	68 646	68 655
486	68 664	68 673	68 681	68 690	68 699	68 708	68 717	68 726	68 735	68 744
487	68 753	68 762	68 771	68 780	68 789	68 797	68 806	68 815	68 824	68 833
488	68 842	68 851	68 860	68 869	68 878	68 886	68 895	68 904	68 913	68 922
489	68 931	68 940	68 949	68 958	68 966	68 975	68 984	68 993	69 002	69 011
490	69 020	69 028	69 037	69 046	69 055	69 064	69 073	69 082	69 090	69 099
491	69 108	69 117	69 126	69 135	69 144	69 152	69 161	69 170	69 179	69 188
492	69 197	69 205	69 214	69 223	69 232	69 241	69 249	69 258	69 267	69 276
493	69 285	69 294	69 302	69 311	69 320	69 329	69 338	69 346	69 355	69 364
494	69 373	69 381	69 390	69 399	69 408	69 417	69 425	69 434	69 443	69 452
495	69 461	69 469	69 478	69 487	69 496	69 504	69 513	69 522	69 531	69 539
496	69 548	69 557	69 566	69 574	69 583	69 592	69 601	69 609	69 618	69 627
497	69 636	69 644	69 653	69 662	69 671	69 679	69 688	69 697	69 705	69 714
498	69 723	69 732	69 740	69 749	69 758	69 767	69 775	69 784	69 793	69 801
499	69 810	69 819	69 827	69 836	69 845	69 854	69 862	69 871	69 880	69 888
500	69 897	69 906	69 914	69 923	69 932	69 940	69 949	69 958	69 966	69 975
501	69 984	69 992	70 001	70 010	70 018	70 027	70 036	70 044	70 053	70 062
502	70 070	70 079	70 088	70 096	70 105	70 114	70 122	70 131	70 140	70 148
503	70 157	70 165	70 174	70 183	70 191	70 200	70 209	70 217	70 226	70 234
504	70 243	70 252	70 260	70 269	70 278	70 286	70 295	70 303	70 312	70 321
505	70 329	70 338	70 346	70 355	70 364	70 372	70 381	70 389	70 398	70 406
506	70 415	70 424	70 432	70 441	70 449	70 458	70 467	70 475	70 484	70 492
507	70 501	70 509	70 518	70 526	70 535	70 544	70 552	70 561	70 569	70 578
508	70 586	70 595	70 603	70 612	70 621	70 629	70 638	70 646	70 655	70 663
509	70 672	70 680	70 689	70 697	70 706	70 714	70 723	70 731	70 740	70 749
510	70 757	70 766	70 774	70 783	70 791	70 800	70 808	70 817	70 825	70 834
511	70 842	70 851	70 859	70 868	70 876	70 885	70 893	70 902	70 910	70 919
512	70 927	70 935	70 944	70 952	70 961	70 969	70 978	70 986	70 995	71 003
513	71 012	71 020	71 029	71 037	71 046	71 054	71 063	71 071	71 079	71 088
514	71 096	71 105	71 113	71 122	71 130	71 139	71 147	71 155	71 164	71 172
515	71 181	71 189	71 198	71 206	71 214	71 223	71 231	71 240	71 248	71 257
516	71 265	71 273	71 282	71 290	71 299	71 307	71 315	71 324	71 332	71 341
517	71 349	71 357	71 366	71 374	71 383	71 391	71 399	71 408	71 416	71 425
518	71 433	71 441	71 450	71 458	71 466	71 475	71 483	71 492	71 500	71 508
519	71 517	71 525	71 533	71 542	71 550	71 559	71 567	71 575	71 584	71 592

No.	0	1	2	3	4	5	6	7	8	9
520	71 600	71 609	71 617	71 625	71 634	71 642	71 650	71 659	71 667	71 675
521	71 684	71 692	71 700	71 709	71 717	71 725	71 734	71 742	71 750	71 759
522	71 767	71 775	71 784	71 792	71 800	71 809	71 817	71 825	71 834	71 842
523	71 850	71 858	71 867	71 875	71 883	71 892	71 900	71 908	71 917	71 925
524	71 933	71 941	71 950	71 958	71 966	71 975	71 983	71 991	71 999	72 008
525	72 016	72 024	72 032	72 041	72 049	72 057	72 066	72 074	72 082	72 090
526	72 099	72 107	72 115	72 123	72 132	72 140	72 148	72 156	72 165	72 173
527	72 181	72 189	72 198	72 206	72 214	72 222	72 230	72 239	72 247	72 255
528	72 263	72 272	72 280	72 288	72 296	72 304	72 313	72 321	72 329	72 337
529	72 346	72 354	72 362	72 370	72 378	72 387	72 395	72 403	72 411	72 419
530	72 428	72 436	72 444	72 452	72 460	72 469	72 477	72 485	72 493	72 501
531	72 509	72 518	72 526	72 534	72 542	72 550	72 558	72 567	72 575	72 583
532	72 591	72 599	72 607	72 616	72 624	72 632	72 640	72 648	72 656	72 665
533	72 673	72 681	72 689	72 697	72 705	72 713	72 722	72 730	72 738	72 746
534	72 754	72 762	72 770	72 779	72 787	72 795	72 803	72 811	72 819	72 827
535	72 835	72 843	72 852	72 860	72 868	72 876	72 884	72 892	72 900	72 908
536	72 916	72 925	72 933	72 941	72 949	72 957	72 965	72 973	72 981	72 989
537	72 997	73 006	73 014	73 022	73 030	73 038	73 046	73 054	73 062	73 070
538	73 078	73 086	73 094	73 102	73 111	73 119	73 127	73 135	73 143	73 151
539	73 159	73 167	73 175	73 183	73 191	73 199	73 207	73 215	73 223	73 231
540	73 239	73 247	73 255	73 263	73 272	73 280	73 288	73 296	73 304	73 312
541	73 320	73 328	73 336	73 344	73 352	73 360	73 368	73 376	73 384	73 392
542	73 400	73 408	73 416	73 424	73 432	73 440	73 448	73 456	73 464	73 472
543	73 480	73 488	73 496	73 504	73 512	73 520	73 528	73 536	73 544	73 552
544	73 560	73 568	73 576	73 584	73 592	73 600	73 608	73 616	73 624	73 632
545	73 640	73 648	73 656	73 664	73 672	73 679	73 687	73 695	73 703	73 711
546	73 719	73 727	73 735	73 743	73 751	73 759	73 767	73 775	73 783	73 791
547	73 799	73 807	73 815	73 823	73 830	73 838	73 846	73 854	73 862	73 870
548	73 878	73 886	73 894	73 902	73 910	73 918	73 926	73 933	73 941	73 949
549	73 957	73 965	73 973	73 981	73 989	73 997	74 005	74 013	74 020	74 028
550	74 036	74 044	74 052	74 060	74 068	74 076	74 084	74 092	74 099	74 107
551	74 115	74 123	74 131	74 139	74 147	74 155	74 162	74 170	74 178	74 186
552	74 194	74 202	74 210	74 218	74 225	74 233	74 241	74 249	74 257	74 265
553	74 273	74 280	74 288	74 296	74 304	74 312	74 320	74 327	74 335	74 343
554	74 351	74 359	74 367	74 374	74 382	74 390	74 398	74 406	74 414	74 421
555	74 429	74 437	74 445	74 453	74 461	74 468	74 476	74 484	74 492	74 500
556	74 507	74 515	74 523	74 531	74 539	74 547	74 554	74 562	74 570	74 578
557	74 586	74 593	74 601	74 609	74 617	74 624	74 632	74 640	74 648	74 656
558	74 663	74 671	74 679	74 687	74 695	74 702	74 710	74 718	74 726	74 733
559	74 741	74 749	74 757	74 764	74 772	74 780	74 788	74 796	74 803	74 811
560	74 819	74 827	74 834	74 842	74 850	74 858	74 865	74 873	74 881	74 889
561	74 896	74 904	74 912	74 920	74 927	74 935	74 943	74 950	74 958	74 966
562	74 974	74 981	74 989	74 997	75 005	75 012	75 020	75 028	75 035	75 043
563	75 051	75 059	75 066	75 074	75 082	75 089	75 097	75 105	75 113	75 120
564	75 128	75 136	75 143	75 151	75 159	75 166	75 174	75 182	75 189	75 197
565	75 205	75 213	75 220	75 228	75 236	75 243	75 251	75 259	75 266	75 274
566	75 282	75 289	75 297	75 305	75 312	75 320	75 328	75 335	75 343	75 351
567	75 358	75 366	75 374	75 381	75 389	75 397	75 404	75 412	75 420	75 427
568	75 435	75 442	75 450	75 458	75 465	75 473	75 481	75 488	75 496	75 504
569	75 511	75 519	75 526	75 534	75 542	75 549	75 557	75 565	75 572	75 580
570	75 587	75 595	75 603	75 610	75 618	75 626	75 633	75 641	75 648	75 656
571	75 664	75 671	75 679	75 686	75 694	75 702	75 709	75 717	75 724	75 732
572	75 740	75 747	75 755	75 762	75 770	75 778	75 785	75 793	75 800	75 808
573	75 815	75 823	75 831	75 838	75 846	75 853	75 861	75 868	75 876	75 884
574	75 891	75 899	75 906	75 914	75 921	75 929	75 937	75 944	75 952	75 959
575	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
576	76 042	76 050	76 057	76 065	76 072	76 080	76 087	76 095	76 103	76 110
577	76 118	76 125	76 133	76 140	76 148	76 155	76 163	76 170	76 178	76 185
578	76 193	76 200	76 208	76 215	76 223	76 230	76 238	76 245	76 253	76 260
579	76 268	76 275	76 283	76 290	76 298	76 305	76 313	76 320	76 328	76 335

No.	0	1	2	3	4	5	6	7	8	9
580	76 343	76 350	76 358	76 365	76 373	76 380	76 388	76 395	76 403	76 410
581	76 418	76 425	76 433	76 440	76 448	76 455	76 462	76 470	76 477	76 485
582	76 492	76 500	76 507	76 515	76 522	76 530	76 537	76 545	76 552	76 559
583	76 567	76 574	76 582	76 589	76 597	76 604	76 612	76 619	76 626	76 634
584	76 641	76 649	76 656	76 664	76 671	76 678	76 686	76 693	76 701	76 708
585	76 716	76 723	76 730	76 738	76 745	76 753	76 760	76 768	76 775	76 782
586	76 790	76 797	76 805	76 812	76 819	76 827	76 834	76 842	76 849	76 856
587	76 864	76 871	76 879	76 886	76 893	76 901	76 908	76 916	76 923	76 930
588	76 938	76 945	76 953	76 960	76 967	76 975	76 982	76 989	76 997	77 004
589	77 012	77 019	77 026	77 034	77 041	77 048	77 056	77 063	77 070	77 078
590	77 085	77 093	77 100	77 107	77 115	77 122	77 129	77 137	77 144	77 151
591	77 159	77 166	77 173	77 181	77 188	77 195	77 203	77 210	77 217	77 225
592	77 232	77 240	77 247	77 254	77 262	77 269	77 276	77 283	77 291	77 298
593	77 305	77 313	77 320	77 327	77 335	77 342	77 349	77 357	77 364	77 371
594	77 379	77 386	77 393	77 401	77 408	77 415	77 422	77 430	77 437	77 444
595	77 452	77 459	77 466	77 474	77 481	77 488	77 495	77 503	77 510	77 517
596	77 525	77 532	77 539	77 546	77 554	77 561	77 568	77 576	77 583	77 590
597	77 597	77 605	77 612	77 619	77 627	77 634	77 641	77 648	77 656	77 663
598	77 670	77 677	77 685	77 692	77 699	77 706	77 714	77 721	77 728	77 735
599	77 743	77 750	77 757	77 764	77 772	77 779	77 786	77 793	77 801	77 808
600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
601	77 887	77 895	77 902	77 909	77 916	77 924	77 931	77 938	77 945	77 952
602	77 960	77 967	77 974	77 981	77 988	77 996	78 003	78 010	78 017	78 025
603	78 032	78 039	78 046	78 053	78 061	78 068	78 075	78 082	78 089	78 097
604	78 104	78 111	78 118	78 125	78 132	78 140	78 147	78 154	78 161	78 168
605	78 176	78 183	78 190	78 197	78 204	78 211	78 219	78 226	78 233	78 240
606	78 247	78 254	78 262	78 269	78 276	78 283	78 290	78 297	78 305	78 312
607	78 319	78 326	78 333	78 340	78 347	78 355	78 362	78 369	78 376	78 383
608	78 390	78 398	78 405	78 412	78 419	78 426	78 433	78 440	78 447	78 455
609	78 462	78 469	78 476	78 483	78 490	78 497	78 504	78 512	78 519	78 526
610	78 533	78 540	78 547	78 554	78 561	78 569	78 576	78 583	78 590	78 597
611	78 604	78 611	78 618	78 625	78 633	78 640	78 647	78 654	78 661	78 668
612	78 675	78 682	78 689	78 696	78 704	78 711	78 718	78 725	78 732	78 739
613	78 746	78 753	78 760	78 767	78 774	78 781	78 789	78 796	78 803	78 810
614	78 817	78 824	78 831	78 838	78 845	78 852	78 859	78 866	78 873	78 880
615	78 888	78 895	78 902	78 909	78 916	78 923	78 930	78 937	78 944	78 951
616	78 958	78 965	78 972	78 979	78 986	78 993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	79 071	79 078	79 085	79 092
618	79 099	79 106	79 113	79 120	79 127	79 134	79 141	79 148	79 155	79 162
619	79 169	79 176	79 183	79 190	79 197	79 204	79 211	79 218	79 225	79 232
620	79 239	79 246	79 253	79 260	79 267	79 274	79 281	79 288	79 295	79 302
621	79 309	79 316	79 323	79 330	79 337	79 344	79 351	79 358	79 365	79 372
622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
623	79 449	79 456	79 463	79 470	79 477	79 484	79 491	79 498	79 505	79 511
624	79 518	79 525	79 532	79 539	79 546	79 553	79 560	79 567	79 574	79 581
625	79 588	79 595	79 602	79 609	79 616	79 623	79 630	79 637	79 644	79 650
626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
627	79 727	79 734	79 741	79 748	79 754	79 761	79 768	79 775	79 782	79 789
628	79 796	79 803	79 810	79 817	79 824	79 831	79 837	79 844	79 851	79 858
629	79 865	79 872	79 879	79 886	79 893	79 900	79 906	79 913	79 920	79 927
630	79 934	79 941	79 948	79 955	79 962	79 969	79 975	79 982	79 989	79 996
631	80 003	80 010	80 017	80 024	80 030	80 037	80 044	80 051	80 058	80 065
632	80 072	80 079	80 085	80 092	80 099	80 106	80 113	80 120	80 127	80 134
633	80 140	80 147	80 154	80 161	80 168	80 175	80 182	80 188	80 195	80 202
634	80 209	80 216	80 223	80 229	80 236	80 243	80 250	80 257	80 264	80 271
635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
637	80 414	80 421	80 428	80 434	80 441	80 448	80 455	80 462	80 468	80 475
638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
639	80 550	80 557	80 564	80 570	80 577	80 584	80 591	80 598	80 604	80 611

No.	0	1	2	3	4	5	6	7	8	9
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
642	80 754	80 760	80 767	80 774	80 781	80 787	80 794	80 801	80 808	80 814
643	80 821	80 828	80 835	80 841	80 848	80 855	80 862	80 868	80 875	80 882
644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	81 070	81 077	81 084
647	81 090	81 097	81 104	81 111	81 117	81 124	81 131	81 137	81 144	81 151
648	81 158	81 164	81 171	81 178	81 184	81 191	81 198	81 204	81 211	81 218
649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
651	81 358	81 365	81 371	81 378	81 385	81 391	81 398	81 405	81 411	81 418
652	81 425	81 431	81 438	81 445	81 451	81 458	81 465	81 471	81 478	81 485
653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
659	81 889	81 895	81 902	81 908	81 915	81 921	81 928	81 935	81 941	81 948
660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
664	82 217	82 223	82 230	82 236	82 243	82 249	82 256	82 263	82 269	82 276
665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504

No.	0	1	2	3	4	5	6	7	8	9
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076

No.	0	1	2	3	4	5	6	7	8	9
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
797	90 146	90 151	90 157	90 162	90 168	90 173	90 179	90 184	90 189	90 195
798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 249
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	90 363	90 369	90 374	90 380	90 385	90 390	90 396	90 401	90 407	90 412
802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376

No.	0	1	2	3	4	5	6	7	8	9
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	92 054	92 059
833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 075	93 080	93 085	93 090
853	93 095	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
858	93 349	93 354	93 359	93 364	93 369	93 374	93 379	93 384	93 389	93 394
859	93 399	93 404	93 409	93 414	93 420	93 425	93 430	93 435	93 440	93 445
860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	93 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443

No.	0	1	2	3	4	5	6	7	8	9
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308

No.	0	1	2	3	4	5	6	7	8	9
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996

TABLE II.

LOGARITHMIC SINES, COSINES, TANGENTS AND COTANGENTS.

AT INTERVALS

Of 30″ from 0° to 1° and 89° to 90°;
Of 1′ from 1° to 89°

0° to 30′

0°	6 sin	10 cos	6 tan	13 cot	
0′	∞	.00000	∞	∞	**60′**
.5′	.16270	.00000	.16270	.83730	59.5′
1′	.46373	.00000	.46373	.53627	59′
1.5′	.63982	.00000	.63982	.36018	58.5′
2′	.76476	.00000	.76476	.23524	58′
2.5′	.86167	.00000	.86167	.13833	57.5′
3′	**.94085***	.00000	**.94085***	**.05915***	57′
3.5′	.00779	.00000	.00779	.99221	56.5′
4′	.06579	.00000	.06579	.93421	56′
4.5′	.11694	.00000	.11694	.88306	55.5′
5′	.16270	.00000	.16270	.83730	**55′**
5.5′	.20409	.00000	.20409	.79591	54.5′
6′	.24188	.00000	.24188	.75812	54′
6.5′	.27664	.00000	.27664	.72336	53.5′
7′	.30882	.00000	.30882	.69118	53′
7.5′	.33879	.00000	.33879	.66121	52.5′
8′	.36682	.00000	.36682	.63318	52′
8.5′	.39314	.00000	.39314	.60686	51.5′
9′	.41797	.00000	.41797	.58203	51′
9.5′	.44145	.00000	.44145	.55855	50.5′
10′	.46373	.00000	.46373	.53627	**50′**
10.5′	.48491	.00000	.48492	.51508	49.5′
11′	.50512	.00000	.50512	.49488	49′
11.5′	.52442	.00000	.52443	.47557	48.5′
12′	.54291	.00000	.54291	.45709	48′
12.5′	.56064	.00000	.56064	.43936	47.5′
13′	.57767	.00000	.57767	.42233	47′
13.5′	.59406	.00000	.59406	.40594	46.5′
14′	.60985	.00000	.60986	.39014	46′
14.5′	.62509	.00000	.62510	.37490	45.5′
15′	.63982	.00000	.63982	.36018	**45′**
15.5′	.65406	.00000	.65406	.34594	44.5′
16′	.66784	.00000	.66785	.33215	44′
16.5′	.68121	**.00000***	.68121	.31879	43.5′
17′	.69417	.99999	.69418	.30582	43′
17.5′	.70676	.99999	.70677	.29323	42.5′
18′	.71900	.99999	.71900	.28100	42′
18.5′	.73090	.99999	.73090	.26910	41.5′
19′	.74248	.99999	.74248	.25752	41′
19.5′	.75376	.99999	.75377	.24623	40.5′
20′	.76475	.99999	.76476	.23524	**40′**
20.5′	.77548	.99999	.77549	.22451	39.5′
21′	.78594	.99999	.78595	.21405	39′
21.5′	.79616	.99999	.79617	.20383	38.5′
22′	.80615	.99999	.80615	.19385	38′
22.5′	.81591	.99999	.81591	.18409	37.5′
23′	.82545	.99999	.82546	.17454	37′
23.5′	.83479	.99999	.83480	.16520	36.5′
24′	.84393	.99999	.84394	.15606	36′
24.5′	.85289	.99999	.85290	.14710	35.5′
25′	.86166	.99999	.86167	.13833	**35′**
25.5′	.87026	.99999	.87027	.12973	34.5′
26′	.87870	.99999	.87871	.12129	34′
26.5′	.88697	.99999	.88698	.11302	33.5′
27′	.89509	.99999	.89510	.10490	33′
27.5′	.90305	.99999	.90307	.09693	32.5′
28′	.91088	.99999	.91089	.08911	32′
28.5′	.91857	.99999	.91858	.08142	31.5′
29′	.92612	.99998	.92613	.07387	31′
29.5′	.93354	.99998	.93356	.06644	30.5′
30′	.94084	.99998	.94086	.05914	**30′**
	7 cos	9 sin	7 cot	12 tan	89°

89°30′ to 90°

30′ to 1°

0°	7 sin	9 cos	7 tan	12 cot	
30′	.94084	.99998	.94086	.05914	**30′**
30.5′	.94802	.99998	.94804	.05196	29.5′
31′	.95508	.99998	.95510	.04490	29′
31.5′	.96203	.99998	.96205	.03795	28.5′
32′	.96887	.99998	.96889	.03111	28′
32.5′	.97560	.99998	.97562	.02438	27.5′
33′	.98223	.99998	.98225	.01775	27′
33.5′	.98876	.99998	.98878	.01122	26.5′
34′	**.99520***	.99998	**.99522***	**.00478***	26′
34.5′	.00154	.99998	.00156	.99844	25.5′
35′	.00779	.99998	.00781	.99219	**25′**
35.5′	.01395	.99998	.01397	.98603	24.5′
36′	.02002	.99998	.02004	.97996	24′
36.5′	.02601	.99998	.02604	.97396	23.5′
37′	.03192	.99997	.03194	.96806	23′
37.5′	.03775	.99997	.03777	.96223	22.5′
38′	.04350	.99997	.04353	.95647	22′
38.5′	.04918	.99997	.04921	.95079	21.5′
39′	.05478	.99997	.05481	.94519	21′
39.5′	.06031	.99997	.06034	.93966	20.5′
40′	.06578	.99997	.06581	.93419	**20′**
40.5′	.07117	.99997	.07120	.92880	19.5′
41′	.07650	.99997	.07653	.92347	19′
41.5′	.08176	.99997	.08180	.91820	18.5′
42′	.08696	.99997	.08700	.91300	18′
42.5′	.09210	.99997	.09214	.90786	17.5′
43′	.09718	.99997	.09722	.90278	17′
43.5′	.10220	.99997	.10224	.89776	16.5′
44′	.10717	.99996	.10720	.89280	16′
44.5′	.11207	.99996	.11211	.88789	15.5′
45′	.11693	.99996	.11696	.88304	**15′**
45.5′	.12172	.99996	.12176	.87824	14.5′
46′	.12647	.99996	.12651	.87349	14′
46.5′	.13117	.99996	.13121	.86879	13.5′
47′	.13581	.99996	.13585	.86415	13′
47.5′	.14041	.99996	.14045	.85955	12.5′
48′	.14495	.99996	.14500	.85500	12′
48.5′	.14945	.99996	.14950	.85050	11.5′
49′	.15391	.99996	.15395	.84605	11′
49.5′	.15832	.99996	.15836	.84164	10.5′
50′	.16268	.99995	.16273	.83727	**10′**
50.5′	.16700	.99995	.16705	.83295	9.5′
51′	.17128	.99995	.17133	.82867	9′
51.5′	.17552	.99995	.17557	.82443	8.5′
52′	.17971	.99995	.17976	.82024	8′
52.5′	.18387	.99995	.18392	.81608	7.5′
53′	.18798	.99995	.18804	.81196	7′
53.5′	.19206	.99995	.19212	.80788	6.5′
54′	.19610	.99995	.19616	.80384	6′
54.5′	.20010	.99995	.20016	.79984	5.5′
55′	.20407	.99994	.20413	.79587	**5′**
55.5′	.20800	.99994	.20806	.79194	4.5′
56′	.21189	.99994	.21195	.78805	4′
56.5′	.21576	.99994	.21581	.78419	3.5′
57′	.21958	.99994	.21964	.78036	3′
57.5′	.22337	.99994	.22343	.77657	2.5′
58′	.22713	.99994	.22720	.77280	2′
58.5′	.23086	.99994	.23092	.76908	1.5′
59′	.23456	.99994	.23462	.76538	1′
59.5′	.23822	.99993	.23829	.76171	.5′
1°	.24186	.99993	.24192	.75808	**0′**
	8 cos	9 sin	8 cot	11 tan	89°

89° to 89°30′

1°

′	8 sin	9 cos	8 tan	11 cot	′
0	.24186	.99993	.24192	.75808	60
1	.24903	.99993	.24910	.75090	59
2	.25609	.99993	.25616	.74384	58
3	.26304	.99993	.26312	.73688	57
4	.26988	.99992	.26996	.73004	56
5	.27661	.99992	.27669	.72331	55
6	.28324	.99992	.28332	.71668	54
7	.28977	.99992	.28986	.71014	53
8	.29621	.99992	.29629	.70371	52
9	.30255	.99991	.30263	.69737	51
10	.30879	.99991	.30888	.69112	50
11	.31495	.99991	.31505	.68495	49
12	.32103	.99990	.32112	.67888	48
13	.32702	.99990	.32711	.67289	47
14	.33292	.99990	.33302	.66698	46
15	.33875	.99990	.33886	.66114	45
16	.34450	.99989	.34461	.65539	44
17	.35018	.99989	.35029	.64971	43
18	.35578	.99989	.35590	.64410	42
19	.36131	.99989	.36143	.63857	41
20	.36678	.99988	.36689	.63311	40
21	.37217	.99988	.37229	.62771	39
22	.37750	.99988	.37762	.62238	38
23	.38276	.99987	.38289	.61711	37
24	.38796	.99987	.38809	.61191	36
25	.39310	.99987	.39323	.60677	35
26	.39818	.99986	.39832	.60168	34
27	.40320	.99986	.40334	.59666	33
28	.40816	.99986	.40830	.59170	32
29	.41307	.99985	.41321	.58679	31
30	.41792	.99985	.41807	.58193	30
31	.42272	.99985	.42287	.57713	29
32	.42746	.99984	.42762	.57238	28
33	.43216	.99984	.43232	.56768	27
34	.43680	.99984	.43696	.56304	26
35	.44139	.99983	.44156	.55844	25
36	.44594	.99983	.44611	.55389	24
37	.45044	.99983	.45061	.54939	23
38	.45489	.99982	.45507	.54493	22
39	.45930	.99982	.45948	.54052	21
40	.46366	.99982	.46385	.53615	20
41	.46799	.99981	.46817	.53183	19
42	.47226	.99981	.47245	.52755	18
43	.47650	.99981	.47669	.52331	17
44	.48069	.99980	.48089	.51911	16
45	.48485	.99980	.48505	.51495	15
46	.48896	.99979	.48917	.51083	14
47	.49304	.99979	.49325	.50675	13
48	.49708	.99979	.49729	.50271	12
49	.50108	.99978	.50130	.49870	11
50	.50504	.99978	.50527	.49473	10
51	.50897	.99977	.50920	.49080	9
52	.51287	.99977	.51310	.48690	8
53	.51673	.99977	.51696	.48304	7
54	.52055	.99976	.52079	.47921	6
55	.52434	.99976	.52459	.47541	5
56	.52810	.99975	.52835	.47165	4
57	.53183	.99975	.53208	.46792	3
58	.53552	.99974	.53578	.46422	2
59	.53919	.99974	.53945	.46055	1
60	.54282	.99974	.54308	.45692	0
′	8 cos	9 sin	8 cot	11 tan	′

89°

2°

′	8 sin	9 cos	8 tan	11 cot	′
0	.54282	.99974	.54308	.45692	60
1	.54642	.99973	.54669	.45331	59
2	.54999	.99973	.55027	.44973	58
3	.55354	.99972	.55382	.44618	57
4	.55705	.99972	.55734	.44266	56
5	.56054	.99971	.56083	.43917	55
6	.56400	.99971	.56429	.43571	54
7	.56743	.99970	.56773	.43227	53
8	.57084	.99970	.57114	.42886	52
9	.57421	.99969	.57452	.42548	51
10	.57757	.99969	.57788	.42212	50
11	.58089	.99968	.58121	.41879	49
12	.58419	.99968	.58451	.41549	48
13	.58747	.99967	.58779	.41221	47
14	.59072	.99967	.59105	.40895	46
15	.59395	.99967	.59428	.40572	45
16	.59715	.99966	.59749	.40251	44
17	.60033	.99966	.60068	.39932	43
18	.60349	.99965	.60384	.39616	42
19	.60662	.99964	.60698	.39302	41
20	.60973	.99964	.61009	.38991	40
21	.61282	.99963	.61319	.38681	39
22	.61589	.99963	.61626	.38374	38
23	.61894	.99962	.61931	.38069	37
24	.62196	.99962	.62234	.37766	36
25	.62497	.99961	.62535	.37465	35
26	.62795	.99961	.62834	.37166	34
27	.63091	.99960	.63131	.36869	33
28	.63385	.99960	.63426	.36574	32
29	.63678	.99959	.63718	.36282	31
30	.63968	.99959	.64009	.35991	30
31	.64256	.99958	.64298	.35702	29
32	.64543	.99958	.64585	.35415	28
33	.64827	.99957	.64870	.35130	27
34	.65110	.99956	.65154	.34846	26
35	.65391	.99956	.65435	.34565	25
36	.65670	.99955	.65715	.34285	24
37	.65947	.99955	.65993	.34007	23
38	.66223	.99954	.66269	.33731	22
39	.66497	.99954	.66543	.33457	21
40	.66769	.99953	.66816	.33184	20
41	.67039	.99952	.67087	.32913	19
42	.67308	.99952	.67356	.32644	18
43	.67575	.99951	.67624	.32376	17
44	.67841	.99951	.67890	.32110	16
45	.68104	.99950	.68154	.31846	15
46	.68367	.99949	.68417	.31583	14
47	.68627	.99949	.68678	.31322	13
48	.68886	.99948	.68938	.31062	12
49	.69144	.99948	.69196	.30804	11
50	.69400	.99947	.69453	.30547	10
51	.69654	.99946	.69708	.30292	9
52	.69907	.99946	.69962	.30038	8
53	.70159	.99945	.70214	.29786	7
54	.70409	.99944	.70465	.29535	6
55	.70658	.99944	.70714	.29286	5
56	.70905	.99943	.70962	.29038	4
57	.71151	.99942	.71208	.28792	3
58	.71395	.99942	.71453	.28547	2
59	.71638	.99941	.71697	.28303	1
60	.71880	.99940	.71940	.28060	0
′	8 cos	9 sin	8 cot	11 tan	′

88°

3°

′	8 sin	9 cos	8 tan	11 cot	′
0	.71880	.99940	.71940	.28060	60
1	.72120	.99940	.72181	.27819	59
2	.72359	.99939	.72420	.27580	58
3	.72597	.99938	.72659	.27341	57
4	.72834	.99938	.72896	.27104	56
5	.73069	.99937	.73132	.26868	55
6	.73303	.99936	.73366	.26634	54
7	.73535	.99936	.73600	.26400	53
8	.73767	.99935	.73832	.26168	52
9	.73997	.99934	.74063	.25937	51
10	.74226	.99934	.74292	.25708	50
11	.74454	.99933	.74521	.25479	49
12	.74680	.99932	.74748	.25252	48
13	.74906	.99932	.74974	.25026	47
14	.75130	.99931	.75199	.24801	46
15	.75353	.99930	.75423	.24577	45
16	.75575	.99929	.75645	.24355	44
17	.75795	.99929	.75867	.24133	43
18	.76015	.99928	.76087	.23913	42
19	.76234	.99927	.76306	.23694	41
20	.76451	.99926	.76525	.23475	40
21	.76667	.99926	.76742	.23258	39
22	.76883	.99925	.76958	.23042	38
23	.77097	.99924	.77173	.22827	37
24	.77310	.99923	.77387	.22613	36
25	.77522	.99923	.77600	.22400	35
26	.77733	.99922	.77811	.22189	34
27	.77943	.99921	.78022	.21978	33
28	.78152	.99920	.78232	.21768	32
29	.78360	.99920	.78441	.21559	31
30	.78568	.99919	.78649	.21351	30
31	.78774	.99918	.78855	.21145	29
32	.78979	.99917	.79061	.20939	28
33	.79183	.99917	.79266	.20734	27
34	.79386	.99916	.79470	.20530	26
35	.79588	.99915	.79673	.20327	25
36	.79789	.99914	.79875	.20125	24
37	.79990	.99913	.80076	.19924	23
38	.80189	.99913	.80277	.19723	22
39	.80388	.99912	.80476	.19524	21
40	.80585	.99911	.80674	.19326	20
41	.80782	.99910	.80872	.19128	19
42	.80978	.99909	.81068	.18932	18
43	.81173	.99909	.81264	.18736	17
44	.81367	.99908	.81459	.18541	16
45	.81560	.99907	.81653	.18347	15
46	.81752	.99906	.81846	.18154	14
47	.81944	.99905	.82038	.17962	13
48	.82134	.99904	.82230	.17770	12
49	.82324	.99904	.82420	.17580	11
50	.82513	.99903	.82610	.17390	10
51	.82701	.99902	.82799	.17201	9
52	.82888	.99901	.82987	.17013	8
53	.83075	.99900	.83175	.16825	7
54	.83261	.99899	.83361	.16639	6
55	.83446	.99898	.83547	.16453	5
56	.83630	.99898	.83732	.16268	4
57	.83813	.99897	.83916	.16084	3
58	.83996	.99896	.84100	.15900	2
59	.84177	.99895	.84282	.15718	1
60	.84358	.99894	.84464	.15536	0
′	8 cos	9 sin	8 cot	11 tan	′

86°

4°

′	8 sin	9 cos	8 tan	11 cot	′
0	.84358	.99894	.84464	.15536	60
1	.84539	.99893	.84646	.15354	59
2	.84718	.99892	.84826	.15174	58
3	.84897	.99891	.85006	.14994	57
4	.85075	.99891	.85185	.14815	56
5	.85252	.99890	.85363	.14637	55
6	.85429	.99889	.85540	.14460	54
7	.85605	.99888	.85717	.14283	53
8	.85780	.99887	.85893	.14107	52
9	.85955	.99886	.86069	.13931	51
10	.86128	.99885	.86243	.13757	50
11	.86301	.99884	.86417	.13583	49
12	.86474	.99883	.86591	.13409	48
13	.86645	.99882	.86763	.13237	47
14	.86816	.99881	.86935	.13065	46
15	.86987	.99880	.87106	.12894	45
16	.87156	.99879	.87277	.12723	44
17	.87325	.99879	.87447	.12553	43
18	.87494	.99878	.87616	.12384	42
19	.87661	.99877	.87785	.12215	41
20	.87829	.99876	.87953	.12047	40
21	.87995	.99875	.88120	.11880	39
22	.88161	.99874	.88287	.11713	38
23	.88326	.99873	.88453	.11547	37
24	.88490	.99872	.88618	.11382	36
25	.88654	.99871	.88783	.11217	35
26	.88817	.99870	.88948	.11052	34
27	.88980	.99869	.89111	.10889	33
28	.89142	.99868	.89274	.10726	32
29	.89304	.99867	.89437	.10563	31
30	.89464	.99866	.89598	.10402	30
31	.89625	.99865	.89760	.10240	29
32	.89784	.99864	.89920	.10080	28
33	.89943	.99863	.90080	.09920	27
34	.90102	.99862	.90240	.09760	26
35	.90260	.99861	.90399	.09601	25
36	.90417	.99860	.90557	.09443	24
37	.90574	.99859	.90715	.09285	23
38	.90730	.99858	.90872	.09128	22
39	.90885	.99857	.91029	.08971	21
40	.91040	.99856	.91185	.08815	20
41	.91195	.99855	.91340	.08660	19
42	.91349	.99854	.91495	.08505	18
43	.91502	.99853	.91650	.08350	17
44	.91655	.99852	.91803	.08197	16
45	.91807	.99851	.91957	.08043	15
46	.91959	.99850	.92110	.07890	14
47	.92110	.99848	.92262	.07738	13
48	.92261	.99847	.92414	.07586	12
49	.92411	.99846	.92565	.07435	11
50	.92561	.99845	.92716	.07284	10
51	.92710	.99844	.92866	.07134	9
52	.92859	.99843	.93016	.06984	8
53	.93007	.99842	.93165	.06835	7
54	.93154	.99841	.93313	.06687	6
55	.93301	.99840	.93462	.06538	5
56	.93448	.99839	.93609	.06391	4
57	.93594	.99838	.93756	.06244	3
58	.93740	.99837	.93903	.06097	2
59	.93885	.99836	.94049	.05951	1
60	.94030	.99834	.94195	.05805	0
′	8 cos	9 sin	8 cot	11 tan	′

85°

5°

′	8 sin	9 cos	8 tan	11 cot	′
0	.94030	.99834	.94195	.05805	**60**
1	.94174	.99833	.94340	.05660	59
2	.94317	.99832	.94485	.05515	58
3	.94461	.99831	.94630	.05370	57
4	.94603	.99830	.94773	.05227	56
5	.94746	.99829	.94917	.05083	55
6	.94887	.99828	.95060	.04940	54
7	.95029	.99827	.95202	.04798	53
8	.95170	.99825	.95344	.04656	52
9	.95310	.99824	.95486	.04514	51
10	.95450	.99823	.95627	.04373	**50**
11	.95589	.99822	.95767	.04233	49
12	.95728	.99821	.95908	.04092	48
13	.95867	.99820	.96047	.03953	47
14	.96005	.99819	.96187	.03813	46
15	.96143	.99817	.96325	.03675	45
16	.96280	.99816	.96464	.03536	44
17	.96417	.99815	.96602	.03398	43
18	.96553	.99814	.96739	.03261	42
19	.96689	.99813	.96877	.03123	41
20	.96825	.99812	.97013	.02987	**40**
21	.96960	.99810	.97150	.02850	39
22	.97095	.99809	.97285	.02715	38
23	.97229	.99808	.97421	.02579	37
24	.97363	.99807	.97556	.02444	36
25	.97496	.99806	.97691	.02309	35
26	.97629	.99804	.97825	.02175	34
27	.97762	.99803	.97959	.02041	33
28	.97894	.99802	.98092	.01908	32
29	.98026	.99801	.98225	.01775	31
30	.98157	.99800	.98358	.01642	**30**
31	.98288	.99798	.98490	.01510	29
32	.98419	.99797	.98622	.01378	28
33	.98549	.99796	.98753	.01247	27
34	.98679	.99795	.98884	.01116	26
35	.98808	.99793	.99015	.00985	25
36	.98937	.99792	.99145	.00855	24
37	.99066	.99791	.99275	.00725	23
38	.99194	.99790	.99405	.00595	22
39	.99322	.99788	.99534	.00466	21
40	.99450	.99787	.99662	.00338	**20**
41	.99577	.99786	.99791	.00209	19
42	.99704	.99785	**.99919***	**.00081***	18
43	.99830	.99783	.00046	.99954	17
44	**.99956***	.99782	.00174	.99826	16
45	.00082	.99781	.00301	.99699	15
46	.00207	.99780	.00427	.99573	14
47	.00332	.99778	.00553	.99447	13
48	.00456	.99777	.00679	.99321	12
49	.00581	.99776	.00805	.99195	11
50	.00704	.99775	.00930	.99070	**10**
51	.00828	.99773	.01055	.98945	9
52	.00951	.99772	.01179	.98821	8
53	.01074	.99771	.01303	.98697	7
54	.01196	.99769	.01427	.98573	6
55	.01318	.99768	.01550	.98450	5
56	.01440	.99767	.01673	.98327	4
57	.01561	.99765	.01796	.98204	3
58	.01682	.99764	.01918	.98082	2
59	.01803	.99763	.02040	.97960	1
60	.01923	.99761	.02162	.97838	**0**
′	9 cos	9 sin	9 cot	10 tan	′

84°

6°

′	9 sin	9 cos	9 tan	10 cot	′
0	.01923	.99761	.02162	.97838	**60**
1	.02043	.99760	.02283	.97717	59
2	.02163	.99759	.02404	.97596	58
3	.02283	.99757	.02525	.97475	57
4	.02402	.99756	.02645	.97355	56
5	.02520	.99755	.02766	.97234	55
6	.02639	.99753	.02885	.97115	54
7	.02757	.99752	.03005	.96995	53
8	.02874	.99751	.03124	.96876	52
9	.02992	.99749	.03242	.96758	51
10	.03109	.99748	.03361	.96639	**50**
11	.03226	.99747	.03479	.96521	49
12	.03342	.99745	.03597	.96403	48
13	.03458	.99744	.03714	.96286	47
14	.03574	.99742	.03832	.96168	46
15	.03690	.99741	.03948	.96052	45
16	.03805	.99740	.04065	.95935	44
17	.03920	.99738	.04181	.95819	43
18	.04034	.99737	.04297	.95703	42
19	.04149	.99736	.04413	.95587	41
20	.04262	.99734	.04528	.95472	**40**
21	.04376	.99733	.04643	.95357	39
22	.04490	.99731	.04758	.95242	38
23	.04603	.99730	.04873	.95127	37
24	.04715	.99728	.04987	.95013	36
25	.04828	.99727	.05101	.94899	35
26	.04940	.99726	.05214	.94786	34
27	.05052	.99724	.05328	.94672	33
28	.05164	.99723	.05441	.94559	32
29	.05275	.99721	.05553	.94447	31
30	.05386	.99720	.05666	.94334	**30**
31	.05497	.99718	.05778	.94222	29
32	.05607	.99717	.05890	.94110	28
33	.05717	.99716	.06002	.93998	27
34	.05827	.99714	.06113	.93887	26
35	.05937	.99713	.06224	.93776	25
36	.06046	.99711	.06335	.93665	24
37	.06155	.99710	.06445	.93555	23
38	.06264	.99708	.06556	.93444	22
39	.06372	.99707	.06666	.93334	21
40	.06481	.99705	.06775	.93225	**20**
41	.06589	.99704	.06885	.93115	19
42	.06696	.99702	.06994	.93006	18
43	.06804	.99701	.07103	.92897	17
44	.06911	.99699	.07211	.92789	16
45	.07018	.99698	.07320	.92680	15
46	.07124	.99696	.07428	.92572	14
47	.07231	.99695	.07536	.92464	13
48	.07337	.99693	.07643	.92357	12
49	.07442	.99692	.07751	.92249	11
50	.07548	.99690	.07858	.92142	**10**
51	.07653	.99689	.07964	.92036	9
52	.07758	.99687	.08071	.91929	8
53	.07863	.99686	.08177	.91823	7
54	.07968	.99684	.08283	.91717	6
55	.08072	.99683	.08389	.91611	5
56	.08176	.99681	.08495	.91505	4
57	.08280	.99680	.08600	.91400	3
58	.08383	.99678	.08705	.91295	2
59	.08486	.99677	.08810	.91190	1
60	.08589	.99675	.08914	.91086	**0**
′	9 cos	9 sin	9 cot	10 tan	′

83°

	7°						8°				
′	9 sin	9 cos	9 tan	10 cot	′	′	9 sin	9 cos	9 tan	10 cot	′
0	.08589	.99675	.08914	.91086	**60**	**0**	.14356	.99575	.14780	.85220	**60**
1	.08692	.99674	.09019	.90981	59	1	.14445	.99574	.14872	.85128	59
2	.08795	.99672	.09123	.90877	58	2	.14535	.99572	.14963	.85037	58
3	.08897	.99670	.09227	.90773	57	3	.14624	.99570	.15054	.84946	57
4	.08999	.99669	.09330	.90670	56	4	.14714	.99568	.15145	.84855	56
5	.09101	.99667	.09434	.90566	55	5	.14803	.99566	.15236	.84764	55
6	.09202	.99666	.09537	.90463	54	6	.14891	.99565	.15327	.84673	54
7	.09304	.99664	.09640	.90360	53	7	.14980	.99563	.15417	.84583	53
8	.09405	.99663	.09742	.90258	52	8	.15069	.99561	.15508	.84492	52
9	.09506	.99661	.09845	.90155	51	9	.15157	.99559	.15598	.84402	51
10	.09606	.99659	.09947	.90053	**50**	**10**	.15245	.99557	.15688	.84312	**50**
11	.09707	.99658	.10049	.89951	49	11	.15333	.99556	.15777	.84223	49
12	.09807	.99656	.10150	.89850	48	12	.15421	.99554	.15867	.84133	48
13	.09907	.99655	.10252	.89748	47	13	.15508	.99552	.15956	.84044	47
14	.10006	.99653	.10353	.89647	46	14	.15596	.99550	.16046	.83954	46
15	.10106	.99651	.10454	.89546	45	15	.15683	.99548	.16135	.83865	45
16	.10205	.99650	.10555	.89445	44	16	.15770	.99546	.16224	.83776	44
17	.10304	.99648	.10656	.89344	43	17	.15857	.99545	.16312	.83688	43
18	.10402	.99647	.10756	.89244	42	18	.15944	.99543	.16401	.83599	42
19	.10501	.99645	.10856	.89144	41	19	.16030	.99541	.16489	.83511	41
20	.10599	.99643	.10956	.89044	**40**	**20**	.16116	.99539	.16577	.83423	**40**
21	.10697	.99642	.11056	.88944	39	21	.16203	.99537	.16665	.83335	39
22	.10795	.99640	.11155	.88845	38	22	.16289	.99535	.16753	.83247	38
23	.10893	.99638	.11254	.88746	37	23	.16374	.99533	.16841	.83159	37
24	.10990	.99637	.11353	.88647	36	24	.16460	.99532	.16928	.83072	36
25	.11087	.99635	.11452	.88548	35	25	.16545	.99530	.17016	.82984	35
26	.11184	.99633	.11551	.88449	34	26	.16631	.99528	.17103	.82897	34
27	.11281	.99632	.11649	.88351	33	27	.16716	.99526	.17190	.82810	33
28	.11377	.99630	.11747	.88253	32	28	.16801	.99524	.17277	.82723	32
29	.11474	.99629	.11845	.88155	31	29	.16886	.99522	.17363	.82637	31
30	.11570	.99627	.11943	.88057	**30**	**30**	.16970	.99520	.17450	.82550	**30**
31	.11666	.99625	.12040	.87960	29	31	.17055	.99518	.17536	.82464	29
32	.11761	.99624	.12138	.87862	28	32	.17139	.99517	.17622	.82378	28
33	.11857	.99622	.12235	.87765	27	33	.17223	.99515	.17708	.82292	27
34	.11952	.99620	.12332	.87668	26	34	.17307	.99513	.17794	.82206	26
35	.12047	.99618	.12428	.87572	25	35	.17391	.99511	.17880	.82120	25
36	.12142	.99617	.12525	.87475	24	36	.17474	.99509	.17965	.82035	24
37	.12236	.99615	.12621	.87379	23	37	.17558	.99507	.18051	.81949	23
38	.12331	.99613	.12717	.87283	22	38	.17641	.99505	.18136	.81864	22
39	.12425	.99612	.12813	.87187	21	39	.17724	.99503	.18221	.81779	21
40	.12519	.99610	.12909	.87091	**20**	**40**	.17807	.99501	.18306	.81694	**20**
41	.12612	.99608	.13004	.86996	19	41	.17890	.99499	.18391	.81609	19
42	.12706	.99607	.13099	.86901	18	42	.17973	.99497	.18475	.81525	18
43	.12799	.99605	.13194	.86806	17	43	.18055	.99495	.18560	.81440	17
44	.12892	.99603	.13289	.86711	16	44	.18137	.99494	.18644	.81356	16
45	.12985	.99601	.13384	.86616	15	45	.18220	.99492	.18728	.81272	15
46	.13078	.99600	.13478	.86522	14	46	.18302	.99490	.18812	.81188	14
47	.13171	.99598	.13573	.86427	13	47	.18383	.99488	.18896	.81104	13
48	.13263	.99596	.13667	.86333	12	48	.18465	.99486	.18979	.81021	12
49	.13355	.99595	.13761	.86239	11	49	.18547	.99484	.19063	.80937	11
50	.13447	.99593	.13854	.86146	**10**	**50**	.18628	.99482	.19146	.80854	**10**
51	.13539	.99591	.13948	.86052	9	51	.18709	.99480	.19229	.80771	9
52	.13630	.99589	.14041	.85959	8	52	.18790	.99478	.19312	.80688	8
53	.13722	.99588	.14134	.85866	7	53	.18871	.99476	.19395	.80605	7
54	.13813	.99586	.14227	.85773	6	54	.18952	.99474	.19478	.80522	6
55	.13904	.99584	.14320	.85680	5	55	.19033	.99472	.19561	.80439	5
56	.13994	.99582	.14412	.85588	4	56	.19113	.99470	.19643	.80357	4
57	.14085	.99581	.14504	.85496	3	57	.19193	.99468	.19725	.80275	3
58	.14175	.99579	.14597	.85403	2	58	.19273	.99466	.19807	.80193	2
59	.14266	.99577	.14688	.85312	1	59	.19353	.99464	.19889	.80111	1
60	.14356	.99575	.14780	.85220	**0**	**60**	.19433	.99462	.19971	.80029	**0**
′	9 cos	9 sin	9 cot	10 tan	′	′	9 cos	9 sin	9 cot	10 tan	′
	82°						81°				

9°

′	9 sin	9 cos	9 tan	10 cot	′
0	.19433	.99462	.19971	.80029	**60**
1	.19513	.99460	.20053	.79947	59
2	.19592	.99458	.20134	.79866	58
3	.19672	.99456	.20216	.79784	57
4	.19751	.99454	.20297	.79703	56
5	.19830	.99452	.20378	.79622	55
6	.19909	.99450	.20459	.79541	54
7	.19988	.99448	.20540	.79460	53
8	.20067	.99446	.20621	.79379	52
9	.20145	.99444	.20701	.79299	51
10	.20223	.99442	.20782	.79218	**50**
11	.20302	.99440	.20862	.79138	49
12	.20380	.99438	.20942	.79058	48
13	.20458	.99436	.21022	.78978	47
14	.20535	.99434	.21102	.78898	46
15	.20613	.99432	.21182	.78818	45
16	.20691	.99429	.21261	.78739	44
17	.20768	.99427	.21341	.78659	43
18	.20845	.99425	.21420	.78580	42
19	.20922	.99423	.21499	.78501	41
20	.20999	.99421	.21578	.78422	**40**
21	.21076	.99419	.21657	.78343	39
22	.21153	.99417	.21736	.78264	38
23	.21229	.99415	.21814	.78186	37
24	.21306	.99413	.21893	.78107	36
25	.21382	.99411	.21971	.78029	35
26	.21458	.99409	.22049	.77951	34
27	.21534	.99407	.22127	.77873	33
28	.21610	.99404	.22205	.77795	32
29	.21685	.99402	.22283	.77717	31
30	.21761	.99400	.22361	.77639	**30**
31	.21836	.99398	.22438	.77562	29
32	.21912	.99396	.22516	.77484	28
33	.21987	.99394	.22593	.77407	27
34	.22062	.99392	.22670	.77330	26
35	.22137	.99390	.22747	.77253	25
36	.22211	.99388	.22824	.77176	24
37	.22286	.99385	.22901	.77099	23
38	.22361	.99383	.22977	.77023	22
39	.22435	.99381	.23054	.76946	21
40	.22509	.99379	.23130	.76870	**20**
41	.22583	.99377	.23206	.76794	19
42	.22657	.99375	.23283	.76717	18
43	.22731	.99372	.23359	.76641	17
44	.22805	.99370	.23435	.76565	16
45	.22878	.99368	.23510	.76490	15
46	.22952	.99366	.23586	.76414	14
47	.23025	.99364	.23661	.76339	13
48	.23098	.99362	.23737	.76263	12
49	.23171	.99359	.23812	.76188	11
50	.23244	.99357	.23887	.76113	**10**
51	.23317	.99355	.23962	.76038	9
52	.23390	.99353	.24037	.75963	8
53	.23462	.99351	.24112	.75888	7
54	.23535	.99348	.24186	.75814	6
55	.23607	.99346	.24261	.75739	5
56	.23679	.99344	.24335	.75665	4
57	.23752	.99342	.24410	.75590	3
58	.23823	.99340	.24484	.75516	2
59	.23895	.99337	.24558	.75442	1
60	.23967	.99335	.24632	.75368	**0**
′	9 cos	9 sin	9 cot	10 tan	′

80°

10°

′	9 sin	9 cos	9 tan	10 cot	′
0	.23967	.99335	.24632	.75368	**60**
1	.24039	.99333	.24706	.75294	59
2	.24110	.99331	.24779	.75221	58
3	.24181	.99328	.24853	.75147	57
4	.24253	.99326	.24926	.75074	56
5	.24324	.99324	.25000	.75000	55
6	.24395	.99322	.25073	.74927	54
7	.24466	.99319	.25146	.74854	53
8	.24536	.99317	.25219	.74781	52
9	.24607	.99315	.25292	.74708	51
10	.24677	.99313	.25365	.74635	**50**
11	.24748	.99310	.25437	.74563	49
12	.24818	.99308	.25510	.74490	48
13	.24888	.99306	.25582	.74418	47
14	.24958	.99304	.25655	.74345	46
15	.25028	.99301	.25727	.74273	45
16	.25098	.99299	.25799	.74201	44
17	.25168	.99297	.25871	.74129	43
18	.25237	.99294	.25943	.74057	42
19	.25307	.99292	.26015	.73985	41
20	.25376	.99290	.26086	.73914	**40**
21	.25445	.99288	.26158	.73842	39
22	.25514	.99285	.26229	.73771	38
23	.25583	.99283	.26301	.73699	37
24	.25652	.99281	.26372	.73628	36
25	.25721	.99278	.26443	.73557	35
26	.25790	.99276	.26514	.73486	34
27	.25858	.99274	.26585	.73415	33
28	.25927	.99271	.26655	.73345	32
29	.25995	.99269	.26726	.73274	31
30	.26063	.99267	.26797	.73203	**30**
31	.26131	.99264	.26867	.73133	29
32	.26199	.99262	.26937	.73063	28
33	.26267	.99260	.27008	.72992	27
34	.26335	.99257	.27078	.72922	26
35	.26403	.99255	.27148	.72852	25
36	.26470	.99252	.27218	.72782	24
37	.26538	.99250	.27288	.72712	23
38	.26605	.99248	.27357	.72643	22
39	.26672	.99245	.27427	.72573	21
40	.26739	.99243	.27496	.72504	**20**
41	.26806	.99241	.27566	.72434	19
42	.26873	.99238	.27635	.72365	18
43	.26940	.99236	.27704	.72296	17
44	.27007	.99233	.27773	.72227	16
45	.27073	.99231	.27842	.72158	15
46	.27140	.99229	.27911	.72089	14
47	.27206	.99226	.27980	.72020	13
48	.27273	.99224	.28049	.71951	12
49	.27339	.99221	.28117	.71883	11
50	.27405	.99219	.28186	.71814	**10**
51	.27471	.99217	.28254	.71746	9
52	.27537	.99214	.28323	.71677	8
53	.27602	.99212	.28391	.71609	7
54	.27668	.99209	.28459	.71541	6
55	.27734	.99207	.28527	.71473	5
56	.27799	.99204	.28595	.71405	4
57	.27864	.99202	.28662	.71338	3
58	.27930	.99200	.28730	.71270	2
59	.27995	.99197	.28798	.71202	1
60	.28060	.99195	.28865	.71135	**0**
′	9 cos	9 sin	9 cot	10 tan	′

79°

11°

′	9 sin	9 cos	9 tan	10 cot	′
0	.28060	.99195	.28865	.71135	60
1	.28125	.99192	.28933	.71067	59
2	.28190	.99190	.29000	.71000	58
3	.28254	.99187	.29067	.70933	57
4	.28319	.99185	.29134	.70866	56
5	.28384	.99182	.29201	.70799	55
6	.28448	.99180	.29268	.70732	54
7	.28512	.99177	.29335	.70665	53
8	.28577	.99175	.29402	.70598	52
9	.28641	.99172	.29468	.70532	51
10	.28705	.99170	.29535	.70465	50
11	.28769	.99167	.29601	.70399	49
12	.28833	.99165	.29668	.70332	48
13	.28896	.99162	.29734	.70266	47
14	.28960	.99160	.29800	.70200	46
15	.29024	.99157	.29866	.70134	45
16	.29087	.99155	.29932	.70068	44
17	.29150	.99152	.29998	.70002	43
18	.29214	.99150	.30064	.69936	42
19	.29277	.99147	.30130	.69870	41
20	.29340	.99145	.30195	.69805	40
21	.29403	.99142	.30261	.69739	39
22	.29466	.99140	.30326	.69674	38
23	.29529	.99137	.30391	.69609	37
24	.29591	.99135	.30457	.69543	36
25	.29654	.99132	.30522	.69478	35
26	.29716	.99130	.30587	.69413	34
27	.29779	.99127	.30652	.69348	33
28	.29841	.99124	.30717	.69283	32
29	.29903	.99122	.30782	.69218	31
30	.29966	.99119	.30846	.69154	30
31	.30028	.99117	.30911	.69089	29
32	.30090	.99114	.30975	.69025	28
33	.30151	.99112	.31040	.68960	27
34	.30213	.99109	.31104	.68896	26
35	.30275	.99106	.31168	.68832	25
36	.30336	.99104	.31233	.68767	24
37	.30398	.99101	.31297	.68703	23
38	.30459	.99099	.31361	.68639	22
39	.30521	.99096	.31425	.68575	21
40	.30582	.99093	.31489	.68511	20
41	.30643	.99091	.31552	.68448	19
42	.30704	.99088	.31616	.68384	18
43	.30765	.99086	.31679	.68321	17
44	.30826	.99083	.31743	.68257	16
45	.30887	.99080	.31806	.68194	15
46	.30947	.99078	.31870	.68130	14
47	.31008	.99075	.31933	.68067	13
48	.31068	.99072	.31996	.68004	12
49	.31129	.99070	.32059	.67941	11
50	.31189	.99067	.32122	.67878	10
51	.31250	.99064	.32185	.67815	9
52	.31310	.99062	.32248	.67752	8
53	.31370	.99059	.32311	.67689	7
54	.31430	.99056	.32373	.67627	6
55	.31490	.99054	.32436	.67564	5
56	.31549	.99051	.32498	.67502	4
57	.31609	.99048	.32561	.67439	3
58	.31669	.99046	.32623	.67377	2
59	.31728	.99043	.32685	.67315	1
60	.31788	.99040	.32747	.67253	0
′	9 cos	9 sin	9 cot	10 tan	′

78°

12°

′	9 sin	9 cos	9 tan	10 cot	′
0	.31788	.99040	.32747	.67253	60
1	.31847	.99038	.32810	.67190	59
2	.31907	.99035	.32872	.67128	58
3	.31966	.99032	.32933	.67067	57
4	.32025	.99030	.32995	.67005	56
5	.32084	.99027	.33057	.66943	55
6	.32143	.99024	.33119	.66881	54
7	.32202	.99022	.33180	.66820	53
8	.32261	.99019	.33242	.66758	52
9	.32319	.99016	.33303	.66697	51
10	.32378	.99013	.33365	.66635	50
11	.32437	.99011	.33426	.66574	49
12	.32495	.99008	.33487	.66513	48
13	.32553	.99005	.33548	.66452	47
14	.32612	.99002	.33609	.66391	46
15	.32670	.99000	.33670	.66330	45
16	.32728	.98997	.33731	.66269	44
17	.32786	.98994	.33792	.66208	43
18	.32844	.98991	.33853	.66147	42
19	.32902	.98989	.33913	.66087	41
20	.32960	.98986	.33974	.66026	40
21	.33018	.98983	.34034	.65966	39
22	.33075	.98980	.34095	.65905	38
23	.33133	.98978	.34155	.65845	37
24	.33190	.98975	.34215	.65785	36
25	.33248	.98972	.34276	.65724	35
26	.33305	.98969	.34336	.65664	34
27	.33362	.98967	.34396	.65604	33
28	.33420	.98964	.34456	.65544	32
29	.33477	.98961	.34516	.65484	31
30	.33534	.98958	.34576	.65424	30
31	.33591	.98955	.34635	.65365	29
32	.33647	.98953	.34695	.65305	28
33	.33704	.98950	.34755	.65245	27
34	.33761	.98947	.34814	.65186	26
35	.33818	.98944	.34874	.65126	25
36	.33874	.98941	.34933	.65067	24
37	.33931	.98938	.34992	.65008	23
38	.33987	.98936	.35051	.64949	22
39	.34043	.98933	.35111	.64889	21
40	.34100	.98930	.35170	.64830	20
41	.34156	.98927	.35229	.64771	19
42	.34212	.98924	.35288	.64712	18
43	.34268	.98921	.35347	.64653	17
44	.34324	.98919	.35405	.64595	16
45	.34380	.98916	.35464	.64536	15
46	.34436	.98913	.35523	.64477	14
47	.34491	.98910	.35581	.64419	13
48	.34547	.98907	.35640	.64360	12
49	.34602	.98904	.35698	.64302	11
50	.34658	.98901	.35757	.64243	10
51	.34713	.98898	.35815	.64185	9
52	.34769	.98896	.35873	.64127	8
53	.34824	.98893	.35931	.64069	7
54	.34879	.98890	.35989	.64011	6
55	.34934	.98887	.36047	.63953	5
56	.34989	.98884	.36105	.63895	4
57	.35044	.98881	.36163	.63837	3
58	.35099	.98878	.36221	.63779	2
59	.35154	.98875	.36279	.63721	1
60	.35209	.98872	.36336	.63664	0
′	9 cos	9 sin	9 cot	10 tan	′

77°

13° 14°

′	9 sin	9 cos	9 tan	10 cot	′	′	9 sin	9 cos	9 tan	10 cot	′
0	.35209	.98872	.36336	.63664	60	0	.38368	.98690	.39677	.60323	60
1	.35263	.98869	.36394	.63606	59	1	.38418	.98687	.39731	.60269	59
2	.35318	.98867	.36452	.63548	58	2	.38469	.98684	.39785	.60215	58
3	.35373	.98864	.36509	.63491	57	3	.38519	.98681	.39838	.60162	57
4	.35427	.98861	.36566	.63434	56	4	.38570	.98678	.39892	.60108	56
5	.35481	.98858	.36624	.63376	55	5	.38620	.98675	.39945	.60055	55
6	.35536	.98855	.36681	.63319	54	6	.38670	.98671	.39999	.60001	54
7	.35590	.98852	.36738	.63262	53	7	.38721	.98668	.40052	.59948	53
8	.35644	.98849	.36795	.63205	52	8	.38771	.98665	.40106	.59894	52
9	.35698	.98846	.36852	.63148	51	9	.38821	.98662	.40159	.59841	51
10	.35752	.98843	.36909	.63091	50	10	.38871	.98659	.40212	.59788	50
11	.35806	.98840	.36966	.63034	49	11	.38921	.98656	.40266	.59734	49
12	.35860	.98837	.37023	.62977	48	12	.38971	.98652	.40319	.59681	48
13	.35914	.98834	.37080	.62920	47	13	.39021	.98649	.40372	.59628	47
14	.35968	.98831	.37137	.62863	46	14	.39071	.98646	.40425	.59575	46
15	.36022	.98828	.37193	.62807	45	15	.39121	.98643	.40478	.59522	45
16	.36075	.98825	.37250	.62750	44	16	.39170	.98640	.40531	.59469	44
17	.36129	.98822	.37306	.62694	43	17	.39220	.98636	.40584	.59416	43
18	.36182	.98819	.37363	.62637	42	18	.39270	.98633	.40636	.59364	42
19	.36236	.98816	.37419	.62581	41	19	.39319	.98630	.40689	.59311	41
20	.36289	.98813	.37476	.62524	40	20	.39369	.98627	.40742	.59258	40
21	.36342	.98810	.37532	.62468	39	21	.39418	.98623	.40795	.59205	39
22	.36395	.98807	.37588	.62412	38	22	.39467	.98620	.40847	.59153	38
23	.36449	.98804	.37644	.62356	37	23	.39517	.98617	.40900	.59100	37
24	.36502	.98801	.37700	.62300	36	24	.39566	.98614	.40952	.59048	36
25	.36555	.98798	.37756	.62244	35	25	.39615	.98610	.41005	.58995	35
26	.36608	.98795	.37812	.62188	34	26	.39664	.98607	.41057	.58943	34
27	.36660	.98792	.37868	.62132	33	27	.39713	.98604	.41109	.58891	33
28	.36713	.98789	.37924	.62076	32	28	.39762	.98601	.41161	.58839	32
29	.36766	.98786	.37980	.62020	31	29	.39811	.98597	.41214	.58786	31
30	.36819	.98783	.38035	.61965	30	30	.39860	.98594	.41266	.58734	30
31	.36871	.98780	.38091	.61909	29	31	.39909	.98591	.41318	.58682	29
32	.36924	.98777	.38147	.61853	28	32	.39958	.98588	.41370	.58630	28
33	.36976	.98774	.38202	.61798	27	33	.40006	.98584	.41422	.58578	27
34	.37028	.98771	.38257	.61743	26	34	.40055	.98581	.41474	.58526	26
35	.37081	.98768	.38313	.61687	25	35	.40103	.98578	.41526	.58474	25
36	.37133	.98765	.38368	.61632	24	36	.40152	.98574	.41578	.58422	24
37	.37185	.98762	.38423	.61577	23	37	.40200	.98571	.41629	.58371	23
38	.37237	.98759	.38479	.61521	22	38	.40249	.98568	.41681	.58319	22
39	.37289	.98756	.38534	.61466	21	39	.40297	.98565	.41733	.58267	21
40	.37341	.98753	.38589	.61411	20	40	.40346	.98561	.41784	.58216	20
41	.37393	.98750	.38644	.61356	19	41	.40394	.98558	.41836	.58164	19
42	.37445	.98746	.38699	.61301	18	42	.40442	.98555	.41887	.58113	18
43	.37497	.98743	.38754	.61246	17	43	.40490	.98551	.41939	.58061	17
44	.37549	.98740	.38808	.61192	16	44	.40538	.98548	.41990	.58010	16
45	.37600	.98737	.38863	.61137	15	45	.40586	.98545	.42041	.57959	15
46	.37652	.98734	.38918	.61082	14	46	.40634	.98541	.42093	.57907	14
47	.37703	.98731	.38972	.61028	13	47	.40682	.98538	.42144	.57856	13
48	.37755	.98728	.39027	.60973	12	48	.40730	.98535	.42195	.57805	12
49	.37806	.98725	.39082	.60918	11	49	.40778	.98531	.42246	.57754	11
50	.37858	.98722	.39136	.60864	10	50	.40825	.98528	.42297	.57703	10
51	.37909	.98719	.39190	.60810	9	51	.40873	.98525	.42348	.57652	9
52	.37960	.98715	.39245	.60755	8	52	.40921	.98521	.42399	.57601	8
53	.38011	.98712	.39299	.60701	7	53	.40968	.98518	.42450	.57550	7
54	.38062	.98709	.39353	.60647	6	54	.41016	.98515	.42501	.57499	6
55	.38113	.98706	.39407	.60593	5	55	.41063	.98511	.42552	.57448	5
56	.38164	.98703	.39461	.60539	4	56	.41111	.98508	.42603	.57397	4
57	.38215	.98700	.39515	.60485	3	57	.41158	.98505	.42653	.57347	3
58	.38266	.98697	.39569	.60431	2	58	.41205	.98501	.42704	.57296	2
59	.38317	.98694	.39623	.60377	1	59	.41252	.98498	.42755	.57245	1
60	.38368	.98690	.39677	.60323	0	60	.41300	.98494	.42805	.57195	0
′	9 cos	9 sin	9 cot	10 tan	′	′	9 cos	9 sin	9 cot	10 tan	′

76° 75°

15°

′	9 sin	9 cos	9 tan	10 cot	′
0	.41300	.98494	.42805	.57195	60
1	.41347	.98491	.42856	.57144	59
2	.41394	.98488	.42906	.57094	58
3	.41441	.98484	.42957	.57043	57
4	.41488	.98481	.43007	.56993	56
5	.41535	.98477	.43057	.56943	55
6	.41582	.98474	.43108	.56892	54
7	.41628	.98471	.43158	.56842	53
8	.41675	.98467	.43208	.56792	52
9	.41722	.98464	.43258	.56742	51
10	.41768	.98460	.43308	.56692	50
11	.41815	.98457	.43358	.56642	49
12	.41861	.98453	.43408	.56592	48
13	.41908	.98450	.43458	.56542	47
14	.41954	.98447	.43508	.56492	46
15	.42001	.98443	.43558	.56442	45
16	.42047	.98440	.43607	.56393	44
17	.42093	.98436	.43657	.56343	43
18	.42140	.98433	.43707	.56293	42
19	.42186	.98429	.43756	.56244	41
20	.42232	.98426	.43806	.56194	40
21	.42278	.98422	.43855	.56145	39
22	.42324	.98419	.43905	.56095	38
23	.42370	.98415	.43954	.56046	37
24	.42416	.98412	.44004	.55996	36
25	.42461	.98409	.44053	.55947	35
26	.42507	.98405	.44102	.55898	34
27	.42553	.98402	.44151	.55849	33
28	.42599	.98398	.44201	.55799	32
29	.42644	.98395	.44250	.55750	31
30	.42690	.98391	.44299	.55701	30
31	.42735	.98388	.44348	.55652	29
32	.42781	.98384	.44397	.55603	28
33	.42826	.98381	.44446	.55554	27
34	.42872	.98377	.44495	.55505	26
35	.42917	.98373	.44544	.55456	25
36	.42962	.98370	.44592	.55408	24
37	.43008	.98366	.44641	.55359	23
38	.43053	.98363	.44690	.55310	22
39	.43098	.98359	.44738	.55262	21
40	.43143	.98356	.44787	.55213	20
41	.43188	.98352	.44836	.55164	19
42	.43233	.98349	.44884	.55116	18
43	.43278	.98345	.44933	.55067	17
44	.43323	.98342	.44981	.55019	16
45	.43367	.98338	.45029	.54971	15
46	.43412	.98334	.45078	.54922	14
47	.43457	.98331	.45126	.54874	13
48	.43502	.98327	.45174	.54826	12
49	.43546	.98324	.45222	.54778	11
50	.43591	.98320	.45271	.54729	10
51	.43635	.98317	.45319	.54681	9
52	.43680	.98313	.45367	.54633	8
53	.43724	.98309	.45415	.54585	7
54	.43769	.98306	.45463	.54537	6
55	.43813	.98302	.45511	.54489	5
56	.43857	.98299	.45559	.54441	4
57	.43901	.98295	.45606	.54394	3
58	.43946	.98291	.45654	.54346	2
59	.43990	.98288	.45702	.54298	1
60	.44034	.98284	.45750	.54250	0
′	9 cos	9 sin	9 cot	10 tan	′

74°

16°

′	9 sin	9 cos	9 tan	10 cot	′
0	.44034	.98284	.45750	.54250	60
1	.44078	.98281	.45797	.54203	59
2	.44122	.98277	.45845	.54155	58
3	.44166	.98273	.45892	.54108	57
4	.44210	.98270	.45940	.54060	56
5	.44253	.98266	.45987	.54013	55
6	.44297	.98262	.46035	.53965	54
7	.44341	.98259	.46082	.53918	53
8	.44385	.98255	.46130	.53870	52
9	.44428	.98251	.46177	.53823	51
10	.44472	.98248	.46224	.53776	50
11	.44516	.98244	.46271	.53729	49
12	.44559	.98240	.46319	.53681	48
13	.44602	.98237	.46366	.53634	47
14	.44646	.98233	.46413	.53587	46
15	.44689	.98229	.46460	.53540	45
16	.44733	.98226	.46507	.53493	44
17	.44776	.98222	.46554	.53446	43
18	.44819	.98218	.46601	.53399	42
19	.44862	.98215	.46648	.53352	41
20	.44905	.98211	.46694	.53306	40
21	.44948	.98207	.46741	.53259	39
22	.44992	.98204	.46788	.53212	38
23	.45035	.98200	.46835	.53165	37
24	.45077	.98196	.46881	.53119	36
25	.45120	.98192	.46928	.53072	35
26	.45163	.98189	.46975	.53025	34
27	.45206	.98185	.47021	.52979	33
28	.45249	.98181	.47068	.52932	32
29	.45292	.98177	.47114	.52886	31
30	.45334	.98174	.47160	.52840	30
31	.45377	.98170	.47207	.52793	29
32	.45419	.98166	.47253	.52747	28
33	.45462	.98162	.47299	.52701	27
34	.45504	.98159	.47346	.52654	26
35	.45547	.98155	.47392	.52608	25
36	.45589	.98151	.47438	.52562	24
37	.45632	.98147	.47484	.52516	23
38	.45674	.98144	.47530	.52470	22
39	.45716	.98140	.47576	.52424	21
40	.45758	.98136	.47622	.52378	20
41	.45801	.98132	.47668	.52332	19
42	.45843	.98129	.47714	.52286	18
43	.45885	.98125	.47760	.52240	17
44	.45927	.98121	.47806	.52194	16
45	.45969	.98117	.47852	.52148	15
46	.46011	.98113	.47897	.52103	14
47	.46053	.98110	.47943	.52057	13
48	.46095	.98106	.47989	.52011	12
49	.46136	.98102	.48035	.51965	11
50	.46178	.98098	.48080	.51920	10
51	.46220	.98094	.48126	.51874	9
52	.46262	.98090	.48171	.51829	8
53	.46303	.98087	.48217	.51783	7
54	.46345	.98083	.48262	.51738	6
55	.46386	.98079	.48307	.51693	5
56	.46428	.98075	.48353	.51647	4
57	.46469	.98071	.48398	.51602	3
58	.46511	.98067	.48443	.51557	2
59	.46552	.98063	.48489	.51511	1
60	.46594	.98060	.48534	.51466	0
′	9 cos	9 sin	9 cot	10 tan	′

73°

17°

′	9 sin	9 cos	9 tan	10 cot	′
0	.46594	.98060	.48534	.51466	60
1	.46635	.98056	.48579	.51421	59
2	.46676	.98052	.48624	.51376	58
3	.46717	.98048	.48669	.51331	57
4	.46758	.98044	.48714	.51286	56
5	.46800	.98040	.48759	.51241	55
6	.46841	.98036	.48804	.51196	54
7	.46882	.98032	.48849	.51151	53
8	.46923	.98029	.48894	.51106	52
9	.46964	.98025	.48939	.51061	51
10	.47005	.98021	.48984	.51016	50
11	.47045	.98017	.49029	.50971	49
12	.47086	.98013	.49073	.50927	48
13	.47127	.98009	.49118	.50882	47
14	.47168	.98005	.49163	.50837	46
15	.47209	.98001	.49207	.50793	45
16	.47249	.97997	.49252	.50748	44
17	.47290	.97993	.49296	.50704	43
18	.47330	.97989	.49341	.50659	42
19	.47371	.97986	.49385	.50615	41
20	.47411	.97982	.49430	.50570	40
21	.47452	.97978	.49474	.50526	39
22	.47492	.97974	.49519	.50481	38
23	.47533	.97970	.49563	.50437	37
24	.47573	.97966	.49607	.50393	36
25	.47613	.97962	.49652	.50348	35
26	.47654	.97958	.49696	.50304	34
27	.47694	.97954	.49740	.50260	33
28	.47734	.97950	.49784	.50216	32
29	.47774	.97946	.49828	.50172	31
30	.47814	.97942	.49872	.50128	30
31	.47854	.97938	.49916	.50084	29
32	.47894	.97934	.49960	.50040	28
33	.47934	.97930	.50004	.49996	27
34	.47974	.97926	.50048	.49952	26
35	.48014	.97922	.50092	.49908	25
36	.48054	.97918	.50136	.49864	24
37	.48094	.97914	.50180	.49820	23
38	.48133	.97910	.50223	.49777	22
39	.48173	.97906	.50267	.49733	21
40	.48213	.97902	.50311	.49689	20
41	.48252	.97898	.50355	.49645	19
42	.48292	.97894	.50398	.49602	18
43	.48332	.97890	.50442	.49558	17
44	.48371	.97886	.50485	.49515	16
45	.48411	.97882	.50529	.49471	15
46	.48450	.97878	.50572	.49428	14
47	.48490	.97874	.50616	.49384	13
48	.48529	.97870	.50659	.49341	12
49	.48568	.97866	.50703	.49297	11
50	.48607	.97861	.50746	.49254	10
51	.48647	.97857	.50789	.49211	9
52	.48686	.97853	.50833	.49167	8
53	.48725	.97849	.50876	.49124	7
54	.48764	.97845	.50919	.49081	6
55	.48803	.97841	.50962	.49038	5
56	.48842	.97837	.51005	.48995	4
57	.48881	.97833	.51048	.48952	3
58	.48920	.97829	.51092	.48908	2
59	.48959	.97825	.51135	.48865	1
60	.48998	.97821	.51178	.48822	0
′	9 cos	9 sin	9 cot	10 tan	′

72°

18°

′	9 sin	9 cos	9 tan	10 cot	′
0	.48998	.97821	.51178	.48822	60
1	.49037	.97817	.51221	.48779	59
2	.49076	.97812	.51264	.48736	58
3	.49115	.97808	.51306	.48694	57
4	.49153	.97804	.51349	.48651	56
5	.49192	.97800	.51392	.48608	55
6	.49231	.97796	.51435	.48565	54
7	.49269	.97792	.51478	.48522	53
8	.49308	.97788	.51520	.48480	52
9	.49347	.97784	.51563	.48437	51
10	.49385	.97779	.51606	.48394	50
11	.49424	.97775	.51648	.48352	49
12	.49462	.97771	.51691	.48309	48
13	.49500	.97767	.51734	.48266	47
14	.49539	.97763	.51776	.48224	46
15	.49577	.97759	.51819	.48181	45
16	.49615	.97754	.51861	.48139	44
17	.49654	.97750	.51903	.48097	43
18	.49692	.97746	.51946	.48054	42
19	.49730	.97742	.51988	.48012	41
20	.49768	.97738	.52031	.47969	40
21	.49806	.97734	.52073	.47927	39
22	.49844	.97729	.52115	.47885	38
23	.49882	.97725	.52157	.47843	37
24	.49920	.97721	.52200	.47800	36
25	.49958	.97717	.52242	.47758	35
26	.49996	.97713	.52284	.47716	34
27	.50034	.97708	.52326	.47674	33
28	.50072	.97704	.52368	.47632	32
29	.50110	.97700	.52410	.47590	31
30	.50148	.97696	.52452	.47548	30
31	.50185	.97691	.52494	.47506	29
32	.50223	.97687	.52536	.47464	28
33	.50261	.97683	.52578	.47422	27
34	.50298	.97679	.52620	.47380	26
35	.50336	.97674	.52661	.47339	25
36	.50374	.97670	.52703	.47297	24
37	.50411	.97666	.52745	.47255	23
38	.50449	.97662	.52787	.47213	22
39	.50486	.97657	.52829	.47171	21
40	.50523	.97653	.52870	.47130	20
41	.50561	.97649	.52912	.47088	19
42	.50598	.97645	.52953	.47047	18
43	.50635	.97640	.52995	.47005	17
44	.50673	.97636	.53037	.46963	16
45	.50710	.97632	.53078	.46922	15
46	.50747	.97628	.53120	.46880	14
47	.50784	.97623	.53161	.46839	13
48	.50821	.97619	.53202	.46798	12
49	.50858	.97615	.53244	.46756	11
50	.50896	.97610	.53285	.46715	10
51	.50933	.97606	.53327	.46673	9
52	.50970	.97602	.53368	.46632	8
53	.51007	.97597	.53409	.46591	7
54	.51043	.97593	.53450	.46550	6
55	.51080	.97589	.53492	.46508	5
56	.51117	.97584	.53533	.46467	4
57	.51154	.97580	.53574	.46426	3
58	.51191	.97576	.53615	.46385	2
59	.51227	.97571	.53656	.46344	1
60	.51264	.97567	.53697	.46303	0
′	9 cos	9 sin	9 cot	10 tan	′

71°

19°

′	9 sin	9 cos	9 tan	10 cot	′
0	.51264	.97567	.53697	.46303	60
1	.51301	.97563	.53738	.46262	59
2	.51338	.97558	.53779	.46221	58
3	.51374	.97554	.53820	.46180	57
4	.51411	.97550	.53861	.46139	56
5	.51447	.97545	.53902	.46098	55
6	.51484	.97541	.53943	.46057	54
7	.51520	.97536	.53984	.46016	53
8	.51557	.97532	.54025	.45975	52
9	.51593	.97528	.54065	.45935	51
10	.51629	.97523	.54106	.45894	50
11	.51666	.97519	.54147	.45853	49
12	.51702	.97515	.54187	.45813	48
13	.51738	.97510	.54228	.45772	47
14	.51774	.97506	.54269	.45731	46
15	.51811	.97501	.54309	.45691	45
16	.51847	.97497	.54350	.45650	44
17	.51883	.97492	.54390	.45610	43
18	.51919	.97488	.54431	.45569	42
19	.51955	.97484	.54471	.45529	41
20	.51991	.97479	.54512	.45488	40
21	.52027	.97475	.54552	.45448	39
22	.52063	.97470	.54593	.45407	38
23	.52099	.97466	.54633	.45367	37
24	.52135	.97461	.54673	.45327	36
25	.52171	.97457	.54714	.45286	35
26	.52207	.97453	.54754	.45246	34
27	.52242	.97448	.54794	.45206	33
28	.52278	.97444	.54835	.45165	32
29	.52314	.97439	.54875	.45125	31
30	.52350	.97435	.54915	.45085	30
31	.52385	.97430	.54955	.45045	29
32	.52421	.97426	.54995	.45005	28
33	.52456	.97421	.55035	.44965	27
34	.52492	.97417	.55075	.44925	26
35	.52527	.97412	.55115	.44885	25
36	.52563	.97408	.55155	.44845	24
37	.52598	.97403	.55195	.44805	23
38	.52634	.97399	.55235	.44765	22
39	.52669	.97394	.55275	.44725	21
40	.52705	.97390	.55315	.44685	20
41	.52740	.97385	.55355	.44645	19
42	.52775	.97381	.55395	.44605	18
43	.52811	.97376	.55434	.44566	17
44	.52846	.97372	.55474	.44526	16
45	.52881	.97367	.55514	.44486	15
46	.52916	.97363	.55554	.44446	14
47	.52951	.97358	.55593	.44407	13
48	.52986	.97353	.55633	.44367	12
49	.53021	.97349	.55673	.44327	11
50	.53056	.97344	.55712	.44288	10
51	.53092	.97340	.55752	.44248	9
52	.53126	.97335	.55791	.44209	8
53	.53161	.97331	.55831	.44169	7
54	.53196	.97326	.55870	.44130	6
55	.53231	.97322	.55910	.44090	5
56	.53266	.97317	.55949	.44051	4
57	.53301	.97312	.55989	.44011	3
58	.53336	.97308	.56028	.43972	2
59	.53370	.97303	.56067	.43933	1
60	.53405	.97299	.56107	.43893	0
′	9 cos	9 sin	9 cot	10 tan	′

70°

20°

′	9 sin	9 cos	9 tan	10 cot	′
0	.53405	.97299	.56107	.43893	60
1	.53440	.97294	.56146	.43854	59
2	.53475	.97289	.56185	.43815	58
3	.53509	.97285	.56224	.43776	57
4	.53544	.97280	.56264	.43736	56
5	.53578	.97276	.56303	.43697	55
6	.53613	.97271	.56342	.43658	54
7	.53647	.97266	.56381	.43619	53
8	.53682	.97262	.56420	.43580	52
9	.53716	.97257	.56459	.43541	51
10	.53751	.97252	.56498	.43502	50
11	.53785	.97248	.56537	.43463	49
12	.53819	.97243	.56576	.43424	48
13	.53854	.97238	.56615	.43385	47
14	.53888	.97234	.56654	.43346	46
15	.53922	.97229	.56693	.43307	45
16	.53957	.97224	.56732	.43268	44
17	.53991	.97220	.56771	.43229	43
18	.54025	.97215	.56810	.43190	42
19	.54059	.97210	.56849	.43151	41
20	.54093	.97206	.56887	.43113	40
21	.54127	.97201	.56926	.43074	39
22	.54161	.97196	.56965	.43035	38
23	.54195	.97192	.57004	.42996	37
24	.54229	.97187	.57042	.42958	36
25	.54263	.97182	.57081	.42919	35
26	.54297	.97178	.57120	.42880	34
27	.54331	.97173	.57158	.42842	33
28	.54365	.97168	.57197	.42803	32
29	.54399	.97163	.57235	.42765	31
30	.54433	.97159	.57274	.42726	30
31	.54466	.97154	.57312	.42688	29
32	.54500	.97149	.57351	.42649	28
33	.54534	.97145	.57389	.42611	27
34	.54567	.97140	.57428	.42572	26
35	.54601	.97135	.57466	.42534	25
36	.54635	.97130	.57504	.42496	24
37	.54668	.97126	.57543	.42457	23
38	.54702	.97121	.57581	.42419	22
39	.54735	.97116	.57619	.42381	21
40	.54769	.97111	.57658	.42342	20
41	.54802	.97107	.57696	.42304	19
42	.54836	.97102	.57734	.42266	18
43	.54869	.97097	.57772	.42228	17
44	.54903	.97092	.57810	.42190	16
45	.54936	.97087	.57849	.42151	15
46	.54969	.97083	.57887	.42113	14
47	.55003	.97078	.57925	.42075	13
48	.55036	.97073	.57963	.42037	12
49	.55069	.97068	.58001	.41999	11
50	.55102	.97063	.58039	.41961	10
51	.55136	.97059	.58077	.41923	9
52	.55169	.97054	.58115	.41885	8
53	.55202	.97049	.58153	.41847	7
54	.55235	.97044	.58191	.41809	6
55	.55268	.97039	.58229	.41771	5
56	.55301	.97035	.58267	.41733	4
57	.55334	.97030	.58304	.41696	3
58	.55367	.97025	.58342	.41658	2
59	.55400	.97020	.58380	.41620	1
60	.55433	.97015	.58418	.41582	0
′	9 cos	9 sin	9 cot	10 tan	′

69°

21°

′	9 sin	9 cos	9 tan	10 cot	′
0	.55433	.97015	.58418	.41582	**60**
1	.55466	.97010	.58455	.41545	59
2	.55499	.97005	.58493	.41507	58
3	.55532	.97001	.58531	.41469	57
4	.55564	.96996	.58569	.41431	56
5	.55597	.96991	.58606	.41394	55
6	.55630	.96986	.58644	.41356	54
7	.55663	.96981	.58681	.41319	53
8	.55695	.96976	.58719	.41281	52
9	.55728	.96971	.58757	.41243	51
10	.55761	.96966	.58794	.41206	**50**
11	.55793	.96962	.58832	.41168	49
12	.55826	.96957	.58869	.41131	48
13	.55858	.96952	.58907	.41093	47
14	.55891	.96947	.58944	.41056	46
15	.55923	.96942	.58981	.41019	45
16	.55956	.96937	.59019	.40981	44
17	.55988	.96932	.59056	.40944	43
18	.56021	.96927	.59094	.40906	42
19	.56053	.96922	.59131	.40869	41
20	.56085	.96917	.59168	.40832	**40**
21	.56118	.96912	.59205	.40795	39
22	.56150	.96907	.59243	.40757	38
23	.56182	.96903	.59280	.40720	37
24	.56215	.96898	.59317	.40683	36
25	.56247	.96893	.59354	.40646	35
26	.56279	.96888	.59391	.40609	34
27	.56311	.96883	.59429	.40571	33
28	.56343	.96878	.59466	.40534	32
29	.56375	.96873	.59503	.40497	31
30	.56408	.96868	.59540	.40460	**30**
31	.56440	.96863	.59577	.40423	29
32	.56472	.96858	.59614	.40386	28
33	.56504	.96853	.59651	.40349	27
34	.56536	.96848	.59688	.40312	26
35	.56568	.96843	.59725	.40275	25
36	.56599	.96838	.59762	.40238	24
37	.56631	.96833	.59799	.40201	23
38	.56663	.96828	.59835	.40165	22
39	.56695	.96823	.59872	.40128	21
40	.56727	.96818	.59909	.40091	**20**
41	.56759	.96813	.59946	.40054	19
42	.56790	.96808	.59983	.40017	18
43	.56822	.96803	.60019	.39981	17
44	.56854	.96798	.60056	.39944	16
45	.56886	.96793	.60093	.39907	15
46	.56917	.96788	.60130	.39870	14
47	.56949	.96783	.60166	.39834	13
48	.56980	.96778	.60203	.39797	12
49	.57012	.96772	.60240	.39760	11
50	.57044	.96767	.60276	.39724	**10**
51	.57075	.96762	.60313	.39687	9
52	.57107	.96757	.60349	.39651	8
53	.57138	.96752	.60386	.39614	7
54	.57169	.96747	.60422	.39578	6
55	.57201	.96742	.60459	.39541	5
56	.57232	.96737	.60495	.39505	4
57	.57264	.96732	.60532	.39468	3
58	.57295	.96727	.60568	.39432	2
59	.57326	.96722	.60605	.39395	1
60	.57358	.96717	.60641	.39359	**0**
′	9 cos	9 sin	9 cot	10 tan	′

68°

22°

′	9 sin	9 cos	9 tan	10 cot	′
0	.57358	.96717	.60641	.39359	**60**
1	.57389	.96711	.60677	.39323	59
2	.57420	.96706	.60714	.39286	58
3	.57451	.96701	.60750	.39250	57
4	.57482	.96696	.60786	.39214	56
5	.57514	.96691	.60823	.39177	55
6	.57545	.96686	.60859	.39141	54
7	.57576	.96681	.60895	.39105	53
8	.57607	.96676	.60931	.39069	52
9	.57638	.96670	.60967	.39033	51
10	.57669	.96665	.61004	.38996	**50**
11	.57700	.96660	.61040	.38960	49
12	.57731	.96655	.61076	.38924	48
13	.57762	.96650	.61112	.38888	47
14	.57793	.96645	.61148	.38852	46
15	.57824	.96640	.61184	.38816	45
16	.57855	.96634	.61220	.38780	44
17	.57885	.96629	.61256	.38744	43
18	.57916	.96624	.61292	.38708	42
19	.57947	.96619	.61328	.38672	41
20	.57978	.96614	.61364	.38636	**40**
21	.58008	.96608	.61400	.38600	39
22	.58039	.96603	.61436	.38564	38
23	.58070	.96598	.61472	.38528	37
24	.58101	.96593	.61508	.38492	36
25	.58131	.96588	.61544	.38456	35
26	.58162	.96582	.61579	.38421	34
27	.58192	.96577	.61615	.38385	33
28	.58223	.96572	.61651	.38349	32
29	.58253	.96567	.61687	.38313	31
30	.58284	.96562	.61722	.38278	**30**
31	.58314	.96556	.61758	.38242	29
32	.58345	.96551	.61794	.38206	28
33	.58375	.96546	.61830	.38170	27
34	.58406	.96541	.61865	.38135	26
35	.58436	.96535	.61901	.38099	25
36	.58467	.96530	.61936	.38064	24
37	.58497	.96525	.61972	.38028	23
38	.58527	.96520	.62008	.37992	22
39	.58557	.96514	.62043	.37957	21
40	.58588	.96509	.62079	.37921	**20**
41	.58618	.96504	.62114	.37886	19
42	.58648	.96498	.62150	.37850	18
43	.58678	.96493	.62185	.37815	17
44	.58709	.96488	.62221	.37779	16
45	.58739	.96483	.62256	.37744	15
46	.58769	.96477	.62292	.37708	14
47	.58799	.96472	.62327	.37673	13
48	.58829	.96467	.62362	.37638	12
49	.58859	.96461	.62398	.37602	11
50	.58889	.96456	.62433	.37567	**10**
51	.58919	.96451	.62468	.37532	9
52	.58949	.96445	.62504	.37496	8
53	.58979	.96440	.62539	.37461	7
54	.59009	.96435	.62574	.37426	6
55	.59039	.96429	.62609	.37391	5
56	.59069	.96424	.62645	.37355	4
57	.59098	.96419	.62680	.37320	3
58	.59128	.96413	.62715	.37285	2
59	.59158	.96408	.62750	.37250	1
60	.59188	.96403	.62785	.37215	**0**
′	9 cos	9 sin	9 cot	10 tan	′

67°

23°

′	9 sin	9 cos	9 tan	10 cot	′
0	.59188	.96403	.62785	.37215	60
1	.59218	.96397	.62820	.37180	59
2	.59247	.96392	.62855	.37145	58
3	.59277	.96387	.62890	.37110	57
4	.59307	.96381	.62926	.37074	56
5	.59336	.96376	.62961	.37039	55
6	.59366	.96370	.62996	.37004	54
7	.59396	.96365	.63031	.36969	53
8	.59425	.96660	.63066	.36934	52
9	.59455	.96354	.63101	.36899	51
10	.59484	.96349	.63135	.36865	50
11	.59514	.96343	.63170	.36830	49
12	.59543	.96338	.63205	.36795	48
13	.59573	.96333	.63240	.36760	47
14	.59602	.96327	.63275	.36725	46
15	.59632	.96322	.63310	.36690	45
16	.59661	.96316	.63345	.36655	44
17	.59690	.96311	.63379	.36621	43
18	.59720	.96305	.63414	.36586	42
19	.59749	.96300	.63449	.36551	41
20	.59778	.96294	.63484	.36516	40
21	.59808	.96289	.63519	.36481	39
22	.59837	.96284	.63553	.36447	38
23	.59866	.96278	.63588	.36412	37
24	.59895	.96273	.63623	.36377	36
25	.59924	.96267	.63657	.36343	35
26	.59954	.96262	.63692	.36308	34
27	.59983	.96256	.63726	.36274	33
28	.60012	.96251	.63761	.36239	32
29	.60041	.96245	.63796	.36204	31
30	.60070	.96240	.63830	.36170	30
31	.60099	.96234	.63865	.36135	29
32	.60128	.96229	.63899	.36101	28
33	.60157	.96223	.63934	.36066	27
34	.60186	.96218	.63968	.36032	26
35	.60215	.96212	.64003	.35997	25
36	.60244	.96207	.64037	.35963	24
37	.60273	.96201	.64072	.35928	23
38	.60302	.96196	.64106	.35894	22
39	.60331	.96190	.64140	.35860	21
40	.60359	.96185	.64175	.35825	20
41	.60388	.96179	.64209	.35791	19
42	.60417	.96174	.64243	.35757	18
43	.60446	.96168	.64278	.35722	17
44	.60474	.96162	.64312	.35688	16
45	.60503	.96157	.64346	.35654	15
46	.60532	.96151	.64381	.35619	14
47	.60561	.96146	.64415	.35585	13
48	.60589	.96140	.64449	.35551	12
49	.60618	.96135	.64483	.35517	11
50	.60646	.96129	.64517	.35483	10
51	.60675	.96123	.64552	.35448	9
52	.60704	.96118	.64586	.35414	8
53	.60732	.96112	.64620	.35380	7
54	.60761	.96107	.64654	.35346	6
55	.60789	.96101	.64688	.35312	5
56	.60818	.96095	.64722	.35278	4
57	.60846	.96090	.64756	.35244	3
58	.60875	.96084	.64790	.35210	2
59	.60903	.96079	.64824	.35176	1
60	.60931	.96073	.64858	.35142	0
′	9 cos	9 sin	9 cot	10 tan	′

66°

24°

′	9 sin	9 cos	9 tan	10 cot	′
0	.60931	.96073	.64858	.35142	60
1	.60960	.96067	.64892	.35108	59
2	.60988	.96062	.64926	.35074	58
3	.61016	.96056	.64960	.35040	57
4	.61045	.96050	.64994	.35006	56
5	.61073	.96045	.65028	.34972	55
6	.61101	.96039	.65062	.34938	54
7	.61129	.96034	.65096	.34904	53
8	.61158	.96028	.65130	.34870	52
9	.61186	.96022	.65164	.34836	51
10	.61214	.96017	.65197	.34803	50
11	.61242	.96011	.65231	.34769	49
12	.61270	.96005	.65265	.34735	48
13	.61298	.96000	.65299	.34701	47
14	.61326	.95994	.65333	.34667	46
15	.61354	.95988	.65366	.34634	45
16	.61382	.95982	.65400	.34600	44
17	.61411	.95977	.65434	.34566	43
18	.61438	.95971	.65467	.34533	42
19	.61466	.95965	.65501	.34499	41
20	.61494	.95960	.65535	.34465	40
21	.61522	.95954	.65568	.34432	39
22	.61550	.95948	.65602	.34398	38
23	.61578	.95942	.65636	.34364	37
24	.61606	.95937	.65669	.34331	36
25	.61634	.95931	.65703	.34297	35
26	.61662	.95925	.65736	.34264	34
27	.61689	.95920	.65770	.34230	33
28	.61717	.95914	.65803	.34197	32
29	.61745	.95908	.65837	.34163	31
30	.61773	.95902	.65870	.34130	30
31	.61800	.95897	.65904	.34096	29
32	.61828	.95891	.65937	.34063	28
33	.61856	.95885	.65971	.34029	27
34	.61883	.95879	.66004	.33996	26
35	.61911	.95873	.66038	.33962	25
36	.61939	.95868	.66071	.33929	24
37	.61966	.95862	.66104	.33896	23
38	.61994	.95856	.66138	.33862	22
39	.62021	.95850	.66171	.33829	21
40	.62049	.95844	.66204	.33796	20
41	.62076	.95839	.66238	.33762	19
42	.62104	.95833	.66271	.33729	18
43	.62131	.95827	.66304	.33696	17
44	.62159	.95821	.66337	.33663	16
45	.62186	.95815	.66371	.33629	15
46	.62214	.95810	.66404	.33596	14
47	.62241	.95804	.66437	.33563	13
48	.62268	.95798	.66470	.33530	12
49	.62296	.95792	.66503	.33497	11
50	.62323	.95786	.66537	.33463	10
51	.62350	.95780	.66570	.33430	9
52	.62377	.95775	.66603	.33397	8
53	.62405	.95769	.66636	.33364	7
54	.62432	.95763	.66669	.33331	6
55	.62459	.95757	.66702	.33298	5
56	.62486	.95751	.66735	.33265	4
57	.62513	.95745	.66768	.33232	3
58	.62541	.95739	.66801	.33199	2
59	.62568	.95733	.66834	.33166	1
60	.62595	.95728	.66867	.33133	0
′	9 cos	9 sin	9 cot	10 tan	′

65°

25°

′	9 sin	9 cos	9 tan	10 cot	′
0	.62595	.95728	.66867	.33133	60
1	.62622	.95722	.66900	.33100	59
2	.62649	.95716	.66933	.33067	58
3	.62676	.95710	.66966	.33034	57
4	.62703	.95704	.66999	.33001	56
5	.62730	.95698	.67032	.32968	55
6	.62757	.95692	.67065	.32935	54
7	.62784	.95686	.67098	.32902	53
8	.62811	.95680	.67131	.32869	52
9	.62838	.95674	.67163	.32837	51
10	.62865	.95668	.67196	.32804	50
11	.62892	.95663	.67229	.32771	49
12	.62918	.95657	.67262	.32738	48
13	.62945	.95651	.67295	.32705	47
14	.62972	.95645	.67327	.32673	46
15	.62999	.95639	.67360	.32640	45
16	.63026	.95633	.67393	.32607	44
17	.63052	.95627	.67426	.32574	43
18	.63079	.95621	.67458	.32542	42
19	.63106	.95615	.67491	.32509	41
20	.63133	.95609	.67524	.32476	40
21	.63159	.95603	.67556	.32444	39
22	.63186	.95597	.67589	.32411	38
23	.63213	.95591	.67622	.32378	37
24	.63239	.95585	.67654	.32346	36
25	.63266	.95579	.67687	.32313	35
26	.63292	.95573	.67719	.32281	34
27	.63319	.95567	.67752	.32248	33
28	.63345	.95561	.67785	.32215	32
29	.63372	.95555	.67817	.32183	31
30	.63398	.95549	.67850	.32150	30
31	.63425	.95543	.67882	.32118	29
32	.63451	.95537	.67915	.32085	28
33	.63478	.95531	.67947	.32053	27
34	.63504	.95525	.67980	.32020	26
35	.63531	.95519	.68012	.31988	25
36	.63557	.95513	.68044	.31956	24
37	.63583	.95507	.68077	.31923	23
38	.63610	.95500	.68109	.31891	22
39	.63636	.95494	.68142	.31858	21
40	.63662	.95488	.68174	.31826	20
41	.63689	.95482	.68206	.31794	19
42	.63715	.95476	.68239	.31761	18
43	.63741	.95470	.68271	.31729	17
44	.63767	.95464	.68303	.31697	16
45	.63794	.95458	.68336	.31664	15
46	.63820	.95452	.68368	.31632	14
47	.63846	.95446	.68400	.31600	13
48	.63872	.95440	.68432	.31568	12
49	.63898	.95434	.68465	.31535	11
50	.63924	.95427	.68497	.31503	10
51	.63950	.95421	.68529	.31471	9
52	.63976	.95415	.68561	.31439	8
53	.64002	.95409	.68593	.31407	7
54	.64028	.95403	.68626	.31374	6
55	.64054	.95397	.68658	.31342	5
56	.64080	.95391	.68690	.31310	4
57	.64106	.95384	.68722	.31278	3
58	.64132	.95378	.68754	.31246	2
59	.64158	.95372	.68786	.31214	1
60	.64184	.95366	.68818	.31182	0
′	9 cos	9 sin	9 cot	10 tan	′

64°

26°

′	9 sin	9 cos	9 tan	10 cot	′
0	.64184	.95366	.68818	.31182	60
1	.64210	.95360	.68850	.31150	59
2	.64236	.95354	.68882	.31118	58
3	.64262	.95348	.68914	.31086	57
4	.64288	.95341	.68946	.31054	56
5	.64313	.95335	.68978	.31022	55
6	.64339	.95329	.69010	.30990	54
7	.64365	.95323	.69042	.30958	53
8	.64391	.95317	.69074	.30926	52
9	.64417	.95310	.69106	.30894	51
10	.64442	.95304	.69138	.30862	50
11	.64468	.95298	.69170	.30830	49
12	.64494	.95292	.69202	.30798	48
13	.64519	.95286	.69234	.30766	47
14	.64545	.95279	.69266	.30734	46
15	.64571	.95273	.69298	.30702	45
16	.64596	.95267	.69329	.30671	44
17	.64622	.95261	.69361	.30639	43
18	.64647	.95254	.69393	.30607	42
19	.64673	.95248	.69425	.30575	41
20	.64698	.95242	.69457	.30543	40
21	.64724	.95236	.69488	.30512	39
22	.64749	.95229	.69520	.30480	38
23	.64775	.95223	.69552	.30448	37
24	.64800	.95217	.69584	.30416	36
25	.64826	.95211	.69615	.30385	35
26	.64851	.95204	.69647	.30353	34
27	.64877	.95198	.69679	.30321	33
28	.64902	.95192	.69710	.30290	32
29	.64927	.95185	.69742	.30258	31
30	.64953	.95179	.69774	.30226	30
31	.64978	.95173	.69805	.30195	29
32	.65003	.95167	.69837	.30163	28
33	.65029	.95160	.69868	.30132	27
34	.65054	.95154	.69900	.30100	26
35	.65079	.95148	.69932	.30068	25
36	.65104	.95141	.69963	.30037	24
37	.65130	.95135	.69995	.30005	23
38	.65155	.95129	.70026	.29974	22
39	.65180	.95122	.70058	.29942	21
40	.65205	.95116	.70089	.29911	20
41	.65230	.95110	.70121	.29879	19
42	.65255	.95103	.70152	.29848	18
43	.65281	.95097	.70184	.29816	17
44	.65306	.95090	.70215	.29785	16
45	.65331	.95084	.70247	.29753	15
46	.65356	.95078	.70278	.29722	14
47	.65381	.95071	.70309	.29691	13
48	.65406	.95065	.70341	.29659	12
49	.65431	.95059	.70372	.29628	11
50	.65456	.95052	.70404	.29596	10
51	.65481	.95046	.70435	.29565	9
52	.65506	.95039	.70466	.29534	8
53	.65531	.95033	.70498	.29502	7
54	.65556	.95027	.70529	.29471	6
55	.65580	.95020	.70560	.29440	5
56	.65605	.95014	.70592	.29408	4
57	.65630	.95007	.70623	.29377	3
58	.65655	.95001	.70654	.29346	2
59	.65680	.94995	.70685	.29315	1
60	.65705	.94988	.70717	.29283	0
′	9 cos	9 sin	9 cot	10 tan	′

63°

27°

′	9 sin	9 cos	9 tan	10 cot	′
0	.65705	.94988	.70717	.29283	60
1	.65729	.94982	.70748	.29252	59
2	.65754	.94975	.70779	.29221	58
3	.65779	.94969	.70810	.29190	57
4	.65804	.94962	.70841	.29159	56
5	.65828	.94956	.70873	.29127	55
6	.65853	.94949	.70904	.29096	54
7	.65878	.94943	.70935	.29065	53
8	.65902	.94936	.70966	.29034	52
9	.65927	.94930	.70997	.29003	51
10	.65952	.94923	.71028	.28972	50
11	.65976	.94917	.71059	.28941	49
12	.66001	.94911	.71090	.28910	48
13	.66025	.94904	.71121	.28879	47
14	.66050	.94898	.71153	.28847	46
15	.66075	.94891	.71184	.28816	45
16	.66099	.94885	.71215	.28785	44
17	.66124	.94878	.71246	.28754	43
18	.66148	.94871	.71277	.28723	42
19	.66173	.94865	.71308	.28692	41
20	.66197	.94858	.71339	.28661	40
21	.66221	.94852	.71370	.28630	39
22	.66246	.94845	.71401	.28599	38
23	.66270	.94839	.71431	.28569	37
24	.66295	.94832	.71462	.28538	36
25	.66319	.94826	.71493	.28507	35
26	.66343	.94819	.71524	.28476	34
27	.66368	.94813	.71555	.28445	33
28	.66392	.94806	.71586	.28414	32
29	.66416	.94799	.71617	.28383	31
30	.66441	.94793	.71648	.28352	30
31	.66465	.94786	.71679	.28321	29
32	.66489	.94780	.71709	.28291	28
33	.66513	.94773	.71740	.28260	27
34	.66537	.94767	.71771	.28229	26
35	.66562	.94760	.71802	.28198	25
36	.66586	.94753	.71833	.28167	24
37	.66610	.94747	.71863	.28137	23
38	.66634	.94740	.71894	.28106	22
39	.66658	.94734	.71925	.28075	21
40	.66682	.94727	.71955	.28045	20
41	.66706	.94720	.71986	.28014	19
42	.66731	.94714	.72017	.27983	18
43	.66755	.94707	.72048	.27952	17
44	.66779	.94700	.72078	.27922	16
45	.66803	.94694	.72109	.27891	15
46	.66827	.94687	.72140	.27860	14
47	.66851	.94680	.72170	.27830	13
48	.66875	.94674	.72201	.27799	12
49	.66899	.94667	.72231	.27769	11
50	.66922	.94660	.72262	.27738	10
51	.66946	.94654	.72293	.27707	9
52	.66970	.94647	.72323	.27677	8
53	.66994	.94640	.72354	.27646	7
54	.67018	.94634	.72384	.27616	6
55	.67042	.94627	.72415	.27585	5
56	.67066	.94620	.72445	.27555	4
57	.67090	.94614	.72476	.27524	3
58	.67113	.94607	.72506	.27494	2
59	.67137	.94600	.72537	.27463	1
60	.67161	.94593	.72567	.27433	0
′	9 cos	9 sin	9 cot	10 tan	′

62°

28°

′	9 sin	9 cos	9 tan	10 cot	′
0	.67161	.94593	.72567	.27433	60
1	.67185	.94587	.72598	.27402	59
2	.67208	.94580	.72628	.27372	58
3	.67232	.94573	.72659	.27341	57
4	.67256	.94567	.72689	.27311	56
5	.67280	.94560	.72720	.27280	55
6	.67303	.94553	.72750	.27250	54
7	.67327	.94546	.72780	.27220	53
8	.67350	.94540	.72811	.27189	52
9	.67374	.94533	.72841	.27159	51
10	.67398	.94526	.72872	.27128	50
11	.67421	.94519	.72902	.27098	49
12	.67445	.94513	.72932	.27068	48
13	.67468	.94506	.72963	.27037	47
14	.67492	.94499	.72993	.27007	46
15	.67515	.94492	.73023	.26977	45
16	.67539	.94485	.73054	.26946	44
17	.67562	.94479	.73084	.26916	43
18	.67586	.94472	.73114	.26886	42
19	.67609	.94465	.73144	.26856	41
20	.67633	.94458	.73175	.26825	40
21	.67656	.94451	.73205	.26795	39
22	.67680	.94445	.73235	.26765	38
23	.67703	.94438	.73265	.26735	37
24	.67726	.94431	.73295	.26705	36
25	.67750	.94424	.73326	.26674	35
26	.67773	.94417	.73356	.26644	34
27	.67796	.94410	.73386	.26614	33
28	.67820	.94404	.73416	.26584	32
29	.67843	.94397	.73446	.26554	31
30	.67866	.94390	.73476	.26524	30
31	.67890	.94383	.73507	.26493	29
32	.67913	.94376	.73537	.26463	28
33	.67936	.94369	.73567	.26433	27
34	.67959	.94362	.73597	.26403	26
35	.67982	.94355	.73627	.26373	25
36	.68006	.94349	.73657	.26343	24
37	.68029	.94342	.73687	.26313	23
38	.68052	.94335	.73717	.26283	22
39	.68075	.94328	.73747	.26253	21
40	.68098	.94321	.73777	.26223	20
41	.68121	.94314	.73807	.26193	19
42	.68144	.94307	.73837	.26163	18
43	.68167	.94300	.73867	.26133	17
44	.68190	.94293	.73897	.26103	16
45	.68213	.94286	.73927	.26073	15
46	.68237	.94279	.73957	.26043	14
47	.68260	.94273	.73987	.26013	13
48	.68283	.94266	.74017	.25983	12
49	.68305	.94259	.74047	.25953	11
50	.68328	.94252	.74077	.25923	10
51	.68351	.94245	.74107	.25893	9
52	.68374	.94238	.74137	.25863	8
53	.68397	.94231	.74166	.25834	7
54	.68420	.94224	.74196	.25804	6
55	.68443	.94217	.74226	.25774	5
56	.68466	.94210	.74256	.25744	4
57	.68489	.94203	.74286	.25714	3
58	.68512	.94196	.74316	.25684	2
59	.68534	.94189	.74345	.25655	1
60	.68557	.94182	.74375	.25625	0
′	9 cos	9 sin	9 cot	10 tan	′

61°

29°

′	9 sin	9 cos	9 tan	10 cot	′
0	.68557	.94182	.74375	.25625	**60**
1	.68580	.94175	.74405	.25595	59
2	.68603	.94168	.74435	.25565	58
3	.68625	.94161	.74465	.25535	57
4	.68648	.94154	.74494	.25506	56
5	.68671	.94147	.74524	.25476	55
6	.68694	.94140	.74554	.25446	54
7	.68716	.94133	.74583	.25417	53
8	.68739	.94126	.74613	.25387	52
9	.68762	.94119	.74643	.25357	51
10	.68784	.94112	.74673	.25327	**50**
11	.68807	.94105	.74702	.25298	49
12	.68829	.94098	.74732	.25268	48
13	.68852	.94090	.74762	.25238	47
14	.68875	.94083	.74791	.25209	46
15	.68897	.94076	.74821	.25179	45
16	.68920	.94069	.74851	.25149	44
17	.68942	.94062	.74880	.25120	43
18	.68965	.94055	.74910	.25090	42
19	.68987	.94048	.74939	.25061	41
20	.69010	.94041	.74969	.25031	**40**
21	.69032	.94034	.74998	.25002	39
22	.69055	.94027	.75028	.24972	38
23	.69077	.94020	.75058	.24942	37
24	.69100	.94012	.75087	.24913	36
25	.69122	.94005	.75117	.24883	35
26	.69144	.93998	.75146	.24854	34
27	.69167	.93991	.75176	.24824	33
28	.69189	.93984	.75205	.24795	32
29	.69212	.93977	.75235	.24765	31
30	.69234	.93970	.75264	.24736	**30**
31	.69256	.93963	.75294	.24706	29
32	.69279	.93955	.75323	.24677	28
33	.69301	.93948	.75353	.24647	27
34	.69323	.93941	.75382	.24618	26
35	.69345	.93934	.75411	.24589	25
36	.69368	.93927	.75441	.24559	24
37	.69390	.93920	.75470	.24530	23
38	.69412	.93912	.75500	.24500	22
39	.69434	.93905	.75529	.24471	21
40	.69456	.93898	.75558	.24442	**20**
41	.69479	.93891	.75588	.24412	19
42	.69501	.93884	.75617	.24383	18
43	.69523	.93876	.75647	.24353	17
44	.69545	.93869	.75676	.24324	16
45	.69567	.93862	.75705	.24295	15
46	.69589	.93855	.75735	.24265	14
47	.69611	.93847	.75764	.24236	13
48	.69633	.93840	.75793	.24207	12
49	.69655	.93833	.75822	.24178	11
50	.69677	.93826	.75852	.24148	**10**
51	.69699	.93819	.75881	.24119	9
52	.69721	.93811	.75910	.24090	8
53	.69743	.93804	.75939	.24061	7
54	.69765	.93797	.75969	.24031	6
55	.69787	.93789	.75998	.24002	5
56	.69809	.93782	.76027	.23973	4
57	.69831	.93775	.76056	.23944	3
58	.69853	.93768	.76086	.23914	2
59	.69875	.93760	.76115	.23885	1
60	.69897	.93753	.76144	.23856	**0**
′	9 cos	9 sin	9 cot	10 tan	′

60°

30°

′	9 sin	9 cos	9 tan	10 cot	′
0	.69897	.93753	.76144	.23856	**60**
1	.69919	.93746	.76173	.23827	59
2	.69941	.93738	.76202	.23798	58
3	.69963	.93731	.76231	.23769	57
4	.69984	.93724	.76261	.23739	56
5	.70006	.93717	.76290	.23710	55
6	.70028	.93709	.76319	.23681	54
7	.70050	.93702	.76348	.23652	53
8	.70072	.93695	.76377	.23623	52
9	.70093	.93687	.76406	.23594	51
10	.70115	.93680	.76435	.23565	**50**
11	.70137	.93673	.76464	.23536	49
12	.70159	.93665	.76493	.23507	48
13	.70180	.93658	.76522	.23478	47
14	.70202	.93650	.76551	.23449	46
15	.70224	.93643	.76580	.23420	45
16	.70245	.93636	.76609	.23391	44
17	.70267	.93628	.76639	.23361	43
18	.70288	.93621	.76668	.23332	42
19	.70310	.93614	.76697	.23303	41
20	.70332	.93606	.76725	.23275	**40**
21	.70353	.93599	.76754	.23246	39
22	.70375	.93591	.76783	.23217	38
23	.70396	.93584	.76812	.23188	37
24	.70418	.93577	.76841	.23159	36
25	.70439	.93569	.76870	.23130	35
26	.70461	.93562	.76899	.23101	34
27	.70482	.93554	.76928	.23072	33
28	.70504	.93547	.76957	.23043	32
29	.70525	.93539	.76986	.23014	31
30	.70547	.93532	.77015	.22985	**30**
31	.70568	.93525	.77044	.22956	29
32	.70590	.93517	.77073	.22927	28
33	.70611	.93510	.77101	.22899	27
34	.70633	.93502	.77130	.22870	26
35	.70654	.93495	.77159	.22841	25
36	.70675	.93487	.77188	.22812	24
37	.70697	.93480	.77217	.22783	23
38	.70718	.93472	.77246	.22754	22
39	.70739	.93465	.77274	.22726	21
40	.70761	.93457	.77303	.22697	**20**
41	.70782	.93450	.77332	.22668	19
42	.70803	.93442	.77361	.22639	18
43	.70824	.93435	.77390	.22610	17
44	.70846	.93427	.77418	.22582	16
45	.70867	.93420	.77447	.22553	15
46	.70888	.93412	.77476	.22524	14
47	.70909	.93405	.77505	.22495	13
48	.70931	.93397	.77533	.22467	12
49	.70952	.93390	.77562	.22438	11
50	.70973	.93382	.77591	.22409	**10**
51	.70994	.93375	.77619	.22381	9
52	.71015	.93367	.77648	.22352	8
53	.71036	.93360	.77677	.22323	7
54	.71058	.93352	.77706	.22294	6
55	.71079	.93344	.77734	.22266	5
56	.71100	.93337	.77763	.22237	4
57	.71121	.93329	.77791	.22209	3
58	.71142	.93322	.77820	.22180	2
59	.71163	.93314	.77849	.22151	1
60	.71184	.93307	.77877	.22123	**0**
′	9 cos	9 sin	9 cot	10 tan	′

59°

31°

′	9 sin	9 cos	9 tan	10 cot	′
0	.71184	.93307	.77877	.22123	**60**
1	.71205	.93299	.77906	.22094	59
2	.71226	.93291	.77935	.22065	58
3	.71247	.93284	.77963	.22037	57
4	.71268	.93276	.77992	.22008	56
5	.71289	.93269	.78020	.21980	55
6	.71310	.93261	.78049	.21951	54
7	.71331	.93253	.78077	.21923	53
8	.71352	.93246	.78106	.21894	52
9	.71373	.93238	.78135	.21865	51
10	.71393	.93230	.78163	.21837	**50**
11	.71414	.93223	.78192	.21808	49
12	.71435	.93215	.78220	.21780	48
13	.71456	.93207	.78249	.21751	47
14	.71477	.93200	.78277	.21723	46
15	.71498	.93192	.78306	.21694	45
16	.71519	.93184	.78334	.21666	44
17	.71539	.93177	.78363	.21637	43
18	.71560	.93169	.78391	.21609	42
19	.71581	.93161	.78419	.21581	41
20	.71602	.93154	.78448	.21552	**40**
21	.71622	.93146	.78476	.21524	39
22	.71643	.93138	.78505	.21495	38
23	.71664	.93131	.78533	.21467	37
24	.71685	.93123	.78562	.21438	36
25	.71705	.93115	.78590	.21410	35
26	.71726	.93108	.78618	.21382	34
27	.71747	.93100	.78647	.21353	33
28	.71767	.93092	.78675	.21325	32
29	.71788	.93084	.78704	.21296	31
30	.71809	.93077	.78732	.21268	**30**
31	.71829	.93069	.78760	.21240	29
32	.71850	.93061	.78789	.21211	28
33	.71870	.93053	.78817	.21183	27
34	.71891	.93046	.78845	.21155	26
35	.71911	.93038	.78874	.21126	25
36	.71932	.93030	.78902	.21098	24
37	.71952	.93022	.78930	.21070	23
38	.71973	.93014	.78959	.21041	22
39	.71994	.93007	.78987	.21013	21
40	.72014	.92999	.79015	.20985	**20**
41	.72034	.92991	.79043	.20957	19
42	.72055	.92983	.79072	.20928	18
43	.72075	.92976	.79100	.20900	17
44	.72096	.92968	.79128	.20872	16
45	.72116	.92960	.79156	.20844	15
46	.72137	.92952	.79185	.20815	14
47	.72157	.92944	.79213	.20787	13
48	.72177	.92936	.79241	.20759	12
49	.72198	.92929	.79269	.20731	11
50	.72218	.92921	.79297	.20703	**10**
51	.72238	.92913	.79326	.20674	9
52	.72259	.92905	.79354	.20646	8
53	.72279	.92897	.79382	.20618	7
54	.72299	.92889	.79410	.20590	6
55	.72320	.92881	.79438	.20562	5
56	.72340	.92874	.79466	.20534	4
57	.72360	.92866	.79495	.20505	3
58	.72381	.92858	.79523	.20477	2
59	.72401	.92850	.79551	.20449	1
60	.72421	.92842	.79579	.20421	**0**
′	9 cos	9 sin	9 cot	10 tan	′

58°

32°

′	9 sin	9 cos	9 tan	10 cot	′
0	.72421	.92842	.79579	.20421	**60**
1	.72441	.92834	.79607	.20393	59
2	.72461	.92826	.79635	.20365	58
3	.72482	.92818	.79663	.20337	57
4	.72502	.92810	.79691	.20309	56
5	.72522	.92803	.79719	.20281	55
6	.72542	.92795	.79747	.20253	54
7	.72562	.92787	.79776	.20224	53
8	.72582	.92779	.79804	.20196	52
9	.72602	.92771	.79832	.20168	51
10	.72622	.92763	.79860	.20140	**50**
11	.72643	.92755	.79888	.20112	49
12	.72663	.92747	.79916	.20084	48
13	.72683	.92739	.79944	.20056	47
14	.72703	.92731	.79972	.20028	46
15	.72723	.92723	.80000	.20000	45
16	.72743	.92715	.80028	.19972	44
17	.72763	.92707	.80056	.19944	43
18	.72783	.92699	.80084	.19916	42
19	.72803	.92691	.80112	.19888	41
20	.72823	.92683	.80140	.19860	**40**
21	.72843	.92675	.80168	.19832	39
22	.72863	.92667	.80195	.19805	38
23	.72883	.92659	.80223	.19777	37
24	.72902	.92651	.80251	.19749	36
25	.72922	.92643	.80279	.19721	35
26	.72942	.92635	.80307	.19693	34
27	.72962	.92627	.80335	.19665	33
28	.72982	.92619	.80363	.19637	32
29	.73002	.92611	.80391	.19609	31
30	.73022	.92603	.80419	.19581	**30**
31	.73041	.92595	.80447	.19553	29
32	.73061	.92587	.80474	.19526	28
33	.73081	.92579	.80502	.19498	27
34	.73101	.92571	.80530	.19470	26
35	.73121	.92563	.80558	.19442	25
36	.73140	.92555	.80586	.19414	24
37	.73160	.92546	.80614	.19386	23
38	.73180	.92538	.80642	.19358	22
39	.73200	.92530	.80669	.19331	21
40	.73219	.92522	.80697	.19303	**20**
41	.73239	.92514	.80725	.19275	19
42	.73259	.92506	.80753	.19247	18
43	.73278	.92498	.80781	.19219	17
44	.73298	.92490	.80808	.19192	16
45	.73318	.92482	.80836	.19164	15
46	.73337	.92473	.80864	.19136	14
47	.73357	.92465	.80892	.19108	13
48	.73377	.92457	.80919	.19081	12
49	.73396	.92449	.80947	.19053	11
50	.73416	.92441	.80975	.19025	**10**
51	.73435	.92433	.81003	.18997	9
52	.73455	.92425	.81030	.18970	8
53	.73474	.92416	.81058	.18942	7
54	.73494	.92408	.81086	.18914	6
55	.73513	.92400	.81113	.18887	5
56	.73533	.92392	.81141	.18859	4
57	.73552	.92384	.81169	.18831	3
58	.73572	.92376	.81196	.18804	2
59	.73591	.92367	.81224	.18776	1
60	.73611	.92359	.81252	.18748	**0**
′	9 cos	9 sin	9 cot	10 tan	′

57°

33°

′	9 sin	9 cos	9 tan	10 cot	′
0	.73611	.92359	.81252	.18748	**60**
1	.73630	.92351	.81279	.18721	59
2	.73650	.92343	.81307	.18693	58
3	.73669	.92335	.81335	.18665	57
4	.73689	.92326	.81362	.18638	56
5	.73708	.92318	.81390	.18610	55
6	.73727	.92310	.81418	.18582	54
7	.73747	.92302	.81445	.18555	53
8	.73766	.92293	.81473	.18527	52
9	.73785	.92285	.81500	.18500	51
10	.73805	.92277	.81528	.18472	**50**
11	.73824	.92269	.81556	.18444	49
12	.73843	.92260	.81583	.18417	48
13	.73863	.92252	.81611	.18389	47
14	.73882	.92244	.81638	.18362	46
15	.73901	.92235	.81666	.18334	45
16	.73921	.92227	.81693	.18307	44
17	.73940	.92219	.81721	.18279	43
18	.73959	.92211	.81748	.18252	42
19	.73978	.92202	.81776	.18224	41
20	.73997	.92194	.81803	.18197	**40**
21	.74017	.92186	.81831	.18169	39
22	.74036	.92177	.81858	.18142	38
23	.74055	.92169	.81886	.18114	37
24	.74074	.92161	.81913	.18087	36
25	.74093	.92152	.81941	.18059	35
26	.74113	.92144	.81968	.18032	34
27	.74132	.92136	.81996	.18004	33
28	.74151	.92127	.82023	.17977	32
29	.74170	.92119	.82051	.17949	31
30	.74189	.92111	.82078	.17922	**30**
31	.74208	.92102	.82106	.17894	29
32	.74227	.92094	.82133	.17867	28
33	.74246	.92086	.82161	.17839	27
34	.74265	.92077	.82188	.17812	26
35	.74284	.92069	.82215	.17785	25
36	.74303	.92060	.82243	.17757	24
37	.74322	.92052	.82270	.17730	23
38	.74341	.92044	.82298	.17702	22
39	.74360	.92035	.82325	.17675	21
40	.74379	.92027	.82352	.17648	**20**
41	.74398	.92018	.82380	.17620	19
42	.74417	.92010	.82407	.17593	18
43	.74436	.92002	.82435	.17565	17
44	.74455	.91993	.82462	.17538	16
45	.74474	.91985	.82489	.17511	15
46	.74493	.91976	.82517	.17483	14
47	.74512	.91968	.82544	.17456	13
48	.74531	.91959	.82571	.17429	12
49	.74549	.91951	.82599	.17401	11
50	.74568	.91942	.82626	.17374	**10**
51	.74587	.91934	.82653	.17347	9
52	.74606	.91925	.82681	.17319	8
53	.74625	.91917	.82708	.17292	7
54	.74644	.91908	.82735	.17265	6
55	.74662	.91900	.82762	.17238	5
56	.74681	.91891	.82790	.17210	4
57	.74700	.91883	.82817	.17183	3
58	.74719	.91874	.82844	.17156	2
59	.74737	.91866	.82871	.17129	1
60	.74756	.91857	.82899	.17101	**0**
′	9 cos	9 sin	9 cot	10 tan	′

56°

34°

′	9 sin	9 cos	9 tan	10 cot	′
0	.74756	.91857	.82899	.17101	**60**
1	.74775	.91849	.82926	.17074	59
2	.74794	.91840	.82953	.17047	58
3	.74812	.91832	.82980	.17020	57
4	.74831	.91823	.83008	.16992	56
5	.74850	.91815	.83035	.16965	55
6	.74868	.91806	.83062	.16938	54
7	.74887	.91798	.83089	.16911	53
8	.74906	.91789	.83117	.16883	52
9	.74924	.91781	.83144	.16856	51
10	.74943	.91772	.83171	.16829	**50**
11	.74961	.91763	.83198	.16802	49
12	.74980	.91755	.83225	.16775	48
13	.74999	.91746	.83252	.16748	47
14	.75017	.91738	.83280	.16720	46
15	.75036	.91729	.83307	.16693	45
16	.75054	.91720	.83334	.16666	44
17	.75073	.91712	.83361	.16639	43
18	.75091	.91703	.83388	.16612	42
19	.75110	.91695	.83415	.16585	41
20	.75128	.91686	.83442	.16558	**40**
21	.75147	.91677	.83470	.16530	39
22	.75165	.91669	.83497	.16503	38
23	.75184	.91660	.83524	.16476	37
24	.75202	.91651	.83551	.16449	36
25	.75221	.91643	.83578	.16422	35
26	.75239	.91634	.83605	.16395	34
27	.75258	.91625	.83632	.16368	33
28	.75276	.91617	.83659	.16341	32
29	.75294	.91608	.83686	.16314	31
30	.75313	.91599	.83713	.16287	**30**
31	.75331	.91591	.83740	.16260	29
32	.75350	.91582	.83768	.16232	28
33	.75368	.91573	.83795	.16205	27
34	.75386	.91565	.83822	.16178	26
35	.75405	.91556	.83849	.16151	25
36	.75423	.91547	.83876	.16124	24
37	.75441	.91538	.83903	.16097	23
38	.75459	.91530	.83930	.16070	22
39	.75478	.91521	.83957	.16043	21
40	.75496	.91512	.83984	.16016	**20**
41	.75514	.91504	.84011	.15989	19
42	.75533	.91495	.84038	.15962	18
43	.75551	.91486	.84065	.15935	17
44	.75569	.91477	.84092	.15908	16
45	.75587	.91469	.84119	.15881	15
46	.75605	.91460	.84146	.15854	14
47	.75624	.91451	.84173	.15827	13
48	.75642	.91442	.84200	.15800	12
49	.75660	.91433	.84227	.15773	11
50	.75678	.91425	.84254	.15746	**10**
51	.75696	.91416	.84280	.15720	9
52	.75714	.91407	.84307	.15693	8
53	.75733	.91398	.84334	.15666	7
54	.75751	.91389	.84361	.15639	6
55	.75769	.91381	.84388	.15612	5
56	.75787	.91372	.84415	.15585	4
57	.75805	.91363	.84442	.15558	3
58	.75823	.91354	.84469	.15531	2
59	.75841	.91345	.84496	.15504	1
60	.75859	.91336	.84523	.15477	**0**
′	9 cos	9 sin	9 cot	10 tan	′

55°

35° 36°

′	9 sin	9 cos	9 tan	10 cot	′	′	9 sin	9 cos	9 tan	10 cot	′
0	.75859	.91336	.84523	.15477	**60**	**0**	.76922	.90796	.86126	.13874	**60**
1	.75877	.91328	.84550	.15450	59	1	.76939	.90787	.86153	.13847	59
2	.75895	.91319	.84576	.15424	58	2	.76957	.90777	.86179	.13821	58
3	.75913	.91310	.84603	.15397	57	3	.76974	.90768	.86206	.13794	57
4	.75931	.91301	.84630	.15370	56	4	.76991	.90759	.86232	.13768	56
5	.75949	.91292	.84657	.15343	55	5	.77009	.90750	.86259	.13741	55
6	.75967	.91283	.84684	.15316	54	6	.77026	.90741	.86285	.13715	54
7	.75985	.91274	.84711	.15289	53	7	.77043	.90731	.86312	.13688	53
8	.76003	.91266	.84738	.15262	52	8	.77061	.90722	.86338	.13662	52
9	.76021	.91257	.84764	.15236	51	9	.77078	.90713	.86365	.13635	51
10	.76039	.91248	.84791	.15209	**50**	**10**	.77095	.90704	.86392	.13608	**50**
11	.76057	.91239	.84818	.15182	49	11	.77112	.90694	.86418	.13582	49
12	.76075	.91230	.84845	.15155	48	12	.77130	.90685	.86445	.13555	48
13	.76093	.91221	.84872	.15128	47	13	.77147	.90676	.86471	.13529	47
14	.76111	.91212	.84899	.15101	46	14	.77164	.90667	.86498	.13502	46
15	.76129	.91203	.84925	.15075	45	15	.77181	.90657	.86524	.13476	45
16	.76146	.91194	.84952	.15048	44	16	.77199	.90648	.86551	.13449	44
17	.76164	.91185	.84979	.15021	43	17	.77216	.90639	.86577	.13423	43
18	.76182	.91176	.85006	.14994	42	18	.77233	.90630	.86603	.13397	42
19	.76200	.91167	.85033	.14967	41	19	.77250	.90620	.86630	.13370	41
20	.76218	.91158	.85059	.14941	**40**	**20**	.77268	.90611	.86656	.13344	**40**
21	.76236	.91149	.85086	.14914	39	21	.77285	.90602	.86683	.13317	39
22	.76253	.91141	.85113	.14887	38	22	.77302	.90592	.86709	.13291	38
23	.76271	.91132	.85140	.14860	37	23	.77319	.90583	.86736	.13264	37
24	.76289	.91123	.85166	.14834	36	24	.77336	.90574	.86762	.13238	36
25	.76307	.91114	.85193	.14807	35	25	.77353	.90565	.86789	.13211	35
26	.76324	.91105	.85220	.14780	34	26	.77370	.90555	.86815	.13185	34
27	.76342	.91096	.85247	.14753	33	27	.77387	.90546	.86842	.13158	33
28	.76360	.91087	.85273	.14727	32	28	.77405	.90537	.86868	.13132	32
29	.76378	.91078	.85300	.14700	31	29	.77422	.90527	.86894	.13106	31
30	.76395	.91069	.85327	.14673	**30**	**30**	.77439	.90518	.86921	.13079	**30**
31	.76413	.91060	.85354	.14646	29	31	.77456	.90509	.86947	.13053	29
32	.76431	.91051	.85380	.14620	28	32	.77473	.90499	.86974	.13026	28
33	.76448	.91042	.85407	.14593	27	33	.77490	.90490	.87000	.13000	27
34	.76466	.91033	.85434	.14566	26	34	.77507	.90480	.87027	.12973	26
35	.76484	.91023	.85460	.14540	25	35	.77524	.90471	.87053	.12947	25
36	.76501	.91014	.85487	.14513	24	36	.77541	.90462	.87079	.12921	24
37	.76519	.91005	.85514	.14486	23	37	.77558	.90452	.87106	.12894	23
38	.76537	.90996	.85540	.14460	22	38	.77575	.90443	.87132	.12868	22
39	.76554	.90987	.85567	.14433	21	39	.77592	.90434	.87158	.12842	21
40	.76572	.90978	.85594	.14406	**20**	**40**	.77609	.90424	.87185	.12815	**20**
41	.76590	.90969	.85620	.14380	19	41	.77626	.90415	.87211	.12789	19
42	.76607	.90960	.85647	.14353	18	42	.77643	.90405	.87238	.12762	18
43	.76625	.90951	.85674	.14326	17	43	.77660	.90396	.87264	.12736	17
44	.76642	.90942	.85700	.14300	16	44	.77677	.90386	.87290	.12710	16
45	.76660	.90933	.85727	.14273	15	45	.77694	.90377	.87317	.12683	15
46	.76677	.90924	.85754	.14246	14	46	.77711	.90368	.87343	.12657	14
47	.76695	.90915	.85780	.14220	13	47	.77728	.90358	.87369	.12631	13
48	.76712	.90906	.85807	.14193	12	48	.77744	.90349	.87396	.12604	12
49	.76730	.90896	.85834	.14166	11	49	.77761	.90339	.87422	.12578	11
50	.76747	.90887	.85860	.14140	**10**	**50**	.77778	.90330	.87448	.12552	**10**
51	.76765	.90878	.85887	.14113	9	51	.77795	.90320	.87475	.12525	9
52	.76782	.90869	.85913	.14087	8	52	.77812	.90311	.87501	.12499	8
53	.76800	.90860	.85940	.14060	7	53	.77829	.90301	.87527	.12473	7
54	.76817	.90851	.85967	.14033	6	54	.77846	.90292	.87554	.12446	6
55	.76835	.90842	.85993	.14007	5	55	.77862	.90282	.87580	.12420	5
56	.76852	.90832	.86020	.13980	4	56	.77879	.90273	.87606	.12394	4
57	.76870	.90823	.86046	.13954	3	57	.77896	.90263	.87633	.12367	3
58	.76887	.90814	.86073	.13927	2	58	.77913	.90254	.87659	.12341	2
59	.76904	.90805	.86100	.13900	1	59	.77930	.90244	.87685	.12315	1
60	.76922	.90796	.86126	.13874	**0**	**60**	.77946	.90235	.87711	.12289	**0**
′	9 cos	9 sin	9 cot	10 tan	′	′	9 cos	9 sin	9 cot	10 tan	′

54° 53°

37°

′	9 sin	9 cos	9 tan	10 cot	′
0	.77946	.90235	.87711	.12289	60
1	.77963	.90225	.87738	.12262	59
2	.77980	.90216	.87764	.12236	58
3	.77997	.90206	.87790	.12210	57
4	.78013	.90197	.87817	.12183	56
5	.78030	.90187	.87843	.12157	55
6	.78047	.90178	.87869	.12131	54
7	.78063	.90168	.87895	.12105	53
8	.78080	.90159	.87922	.12078	52
9	.78097	.90149	.87948	.12052	51
10	.78113	.90139	.87974	.12026	50
11	.78130	.90130	.88000	.12000	49
12	.78147	.90120	.88027	.11973	48
13	.78163	.90111	.88053	.11947	47
14	.78180	.90101	.88079	.11921	46
15	.78197	.90091	.88105	.11895	45
16	.78213	.90082	.88131	.11869	44
17	.78230	.90072	.88158	.11842	43
18	.78246	.90063	.88184	.11816	42
19	.78263	.90053	.88210	.11790	41
20	.78280	.90043	.88236	.11764	40
21	.78296	.90034	.88262	.11738	39
22	.78313	.90024	.88289	.11711	38
23	.78329	.90014	.88315	.11685	37
24	.78346	.90005	.88341	.11659	36
25	.78362	.89995	.88367	.11633	35
26	.78379	.89985	.88393	.11607	34
27	.78395	.89976	.88420	.11580	33
28	.78412	.89966	.88446	.11554	32
29	.78428	.89956	.88472	.11528	31
30	.78445	.89947	.88498	.11502	30
31	.78461	.89937	.88524	.11476	29
32	.78478	.89927	.88550	.11450	28
33	.78494	.89918	.88577	.11423	27
34	.78510	.89908	.88603	.11397	26
35	.78527	.89898	.88629	.11371	25
36	.78543	.89888	.88655	.11345	24
37	.78560	.89879	.88681	.11319	23
38	.78576	.89869	.88707	.11293	22
39	.78592	.89859	.88733	.11267	21
40	.78609	.89849	.88759	.11241	20
41	.78625	.89840	.88786	.11214	19
42	.78642	.89830	.88812	.11188	18
43	.78658	.89820	.88838	.11162	17
44	.78674	.89810	.88864	.11136	16
45	.78691	.89801	.88890	.11110	15
46	.78707	.89791	.88916	.11084	14
47	.78723	.89781	.88942	.11058	13
48	.78739	.89771	.88968	.11032	12
49	.78756	.89761	.88994	.11006	11
50	.78772	.89752	.89020	.10980	10
51	.78788	.89742	.89046	.10954	9
52	.78805	.89732	.89073	.10927	8
53	.78821	.89722	.89099	.10901	7
54	.78837	.89712	.89125	.10875	6
55	.78853	.89702	.89151	.10849	5
56	.78869	.89693	.89177	.10823	4
57	.78886	.89683	.89203	.10797	3
58	.78902	.89673	.89229	.10771	2
59	.78918	.89663	.89255	.10745	1
60	.78934	.89653	.89281	.10719	0
′	9 cos	9 sin	9 cot	10 tan	′

52°

38°

′	9 sin	9 cos	9 tan	10 cot	′
0	.78934	.89653	.89281	.10719	60
1	.78950	.89643	.89307	.10693	59
2	.78967	.89633	.89333	.10667	58
3	.78983	.89624	.89359	.10641	57
4	.78999	.89614	.89385	.10615	56
5	.79015	.89604	.89411	.10589	55
6	.79031	.89594	.89437	.10563	54
7	.79047	.89584	.89463	.10537	53
8	.79063	.89574	.89489	.10511	52
9	.79079	.89564	.89515	.10485	51
10	.79095	.89554	.89541	.10459	50
11	.79111	.89544	.89567	.10433	49
12	.79128	.89534	.89593	.10407	48
13	.79144	.89524	.89619	.10381	47
14	.79160	.89514	.89645	.10355	46
15	.79176	.89504	.89671	.10329	45
16	.79192	.89495	.89697	.10303	44
17	.79208	.89485	.89723	.10277	43
18	.79224	.89475	.89749	.10251	42
19	.79240	.89465	.89775	.10225	41
20	.79256	.89455	.89801	.10199	40
21	.79272	.89445	.89827	.10173	39
22	.79288	.89435	.89853	.10147	38
23	.79304	.89425	.89879	.10121	37
24	.79319	.89415	.89905	.10095	36
25	.79335	.89405	.89931	.10069	35
26	.79351	.89395	.89957	.10043	34
27	.79367	.89385	.89983	.10017	33
28	.79383	.89375	.90009	.09991	32
29	.79399	.89364	.90035	.09965	31
30	.79415	.89354	.90061	.09939	30
31	.79431	.89344	.90086	.09914	29
32	.79447	.89334	.90112	.09888	28
33	.79463	.89324	.90138	.09862	27
34	.79478	.89314	.90164	.09836	26
35	.79494	.89304	.90190	.09810	25
36	.79510	.89294	.90216	.09784	24
37	.79526	.89284	.90242	.09758	23
38	.79542	.89274	.90268	.09732	22
39	.79558	.89264	.90294	.09706	21
40	.79573	.89254	.90320	.09680	20
41	.79589	.89244	.90346	.09654	19
42	.79605	.89233	.90371	.09629	18
43	.79621	.89223	.90397	.09603	17
44	.79636	.89213	.90423	.09577	16
45	.79652	.89203	.90449	.09551	15
46	.79668	.89193	.90475	.09525	14
47	.79684	.89183	.90501	.09499	13
48	.79699	.89173	.90527	.09473	12
49	.79715	.89162	.90553	.09447	11
50	.79731	.89152	.90578	.09422	10
51	.79746	.89142	.90604	.09396	9
52	.79762	.89132	.90630	.09370	8
53	.79778	.89122	.90656	.09344	7
54	.79793	.89112	.90682	.09318	6
55	.79809	.89101	.90708	.09292	5
56	.79825	.89091	.90734	.09266	4
57	.79840	.89081	.90759	.09241	3
58	.79856	.89071	.90785	.09215	2
59	.79872	.89060	.90811	.09189	1
60	.79887	.89050	.90837	.09163	0
′	9 cos	9 sin	9 cot	10 tan	′

51°

39°

′	9 sin	9 cos	9 tan	10 cot	′
0	.79887	.89050	.90837	.09163	**60**
1	.79903	.89040	.90863	.09137	59
2	.79918	.89030	.90889	.09111	58
3	.79934	.89020	.90914	.09086	57
4	.79950	.89009	.90940	.09060	56
5	.79965	.88999	.90966	.09034	55
6	.79981	.88989	.90992	.09008	54
7	.79996	.88978	.91018	.08982	53
8	.80012	.88968	.91043	.08957	52
9	.80027	.88958	.91069	.08931	51
10	.80043	.88948	.91095	.08905	**50**
11	.80058	.88937	.91121	.08879	49
12	.80074	.88927	.91147	.08853	48
13	.80089	.88917	.91172	.08828	47
14	.80105	.88906	.91198	.08802	46
15	.80120	.88896	.91224	.08776	45
16	.80136	.88886	.91250	.08750	44
17	.80151	.88875	.91276	.08724	43
18	.80166	.88865	.91301	.08699	42
19	.80182	.88855	.91327	.08673	41
20	.80197	.88844	.91353	.08647	**40**
21	.80213	.88834	.91379	.08621	39
22	.80228	.88824	.91404	.08596	38
23	.80244	.88813	.91430	.08570	37
24	.80259	.88803	.91456	.08544	36
25	.80274	.88793	.91482	.08518	35
26	.80290	.88782	.91507	.08493	34
27	.80305	.88772	.91533	.08467	33
28	.80320	.88761	.91559	.08441	32
29	.80336	.88751	.91585	.08415	31
30	.80351	.88741	.91610	.08390	**30**
31	.80366	.88730	.91636	.08364	29
32	.80382	.88720	.91662	.08338	28
33	.80397	.88709	.91688	.08312	27
34	.80412	.88699	.91713	.08287	26
35	.80428	.88688	.91739	.08261	25
36	.80443	.88678	.91765	.08235	24
37	.80458	.88668	.91791	.08209	23
38	.80473	.88657	.91816	.08184	22
39	.80489	.88647	.91842	.08158	21
40	.80504	.88636	.91868	.08132	**20**
41	.80519	.88626	.91893	.08107	19
42	.80534	.88615	.91919	.08081	18
43	.80550	.88605	.91945	.08055	17
44	.80565	.88594	.91971	.08029	16
45	.80580	.88584	.91996	.08004	15
46	.80595	.88573	.92022	.07978	14
47	.80610	.88563	.92048	.07952	13
48	.80625	.88552	.92073	.07927	12
49	.80641	.88542	.92099	.07901	11
50	.80656	.88531	.92125	.07875	**10**
51	.80671	.88521	.92150	.07850	9
52	.80686	.88510	.92176	.07824	8
53	.80701	.88499	.92202	.07798	7
54	.80716	.88489	.92227	.07773	6
55	.80731	.88478	.92253	.07747	5
56	.80746	.88468	.92279	.07721	4
57	.80762	.88457	.92304	.07696	3
58	.80777	.88447	.92330	.07670	2
59	.80792	.88436	.92356	.07644	1
60	.80807	.88425	.92381	.07619	**0**
′	9 cos	9 sin	9 cot	10 tan	′

50°

40°

′	9 sin	9 cos	9 tan	10 cot	′
0	.80807	.88425	.92381	.07619	**60**
1	.80822	.88415	.92407	.07593	59
2	.80837	.88404	.92433	.07567	58
3	.80852	.88394	.92458	.07542	57
4	.80867	.88383	.92484	.07516	56
5	.80882	.88372	.92510	.07490	55
6	.80897	.88362	.92535	.07465	54
7	.80912	.88351	.92561	.07439	53
8	.80927	.88340	.92587	.07413	52
9	.80942	.88330	.92612	.07388	51
10	.80957	.88319	.92638	.07362	**50**
11	.80972	.88308	.92663	.07337	49
12	.80987	.88298	.92689	.07311	48
13	.81002	.88287	.92715	.07285	47
14	.81017	.88276	.92740	.07260	46
15	.81032	.88266	.92766	.07234	45
16	.81047	.88255	.92792	.07208	44
17	.81061	.88244	.92817	.07183	43
18	.81076	.88234	.92843	.07157	42
19	.81091	.88223	.92868	.07132	41
20	.81106	.88212	.92894	.07106	**40**
21	.81121	.88201	.92920	.07080	39
22	.81136	.88191	.92945	.07055	38
23	.81151	.88180	.92971	.07029	37
24	.81166	.88169	.92996	.07004	36
25	.81180	.88158	.93022	.06978	35
26	.81195	.88148	.93048	.06952	34
27	.81210	.88137	.93073	.06927	33
28	.81225	.88126	.93099	.06901	32
29	.81240	.88115	.93124	.06876	31
30	.81254	.88105	.93150	.06850	**30**
31	.81269	.88094	.93175	.06825	29
32	.81284	.88083	.93201	.06799	28
33	.81299	.88072	.93227	.06773	27
34	.81314	.88061	.93252	.06748	26
35	.81328	.88051	.93278	.06722	25
36	.81343	.88040	.93303	.06697	24
37	.81358	.88029	.93329	.06671	23
38	.81372	.88018	.93354	.06646	22
39	.81387	.88007	.93380	.06620	21
40	.81402	.87996	.93406	.06594	**20**
41	.81417	.87985	.93431	.06569	19
42	.81431	.87975	.93457	.06543	18
43	.81446	.87964	.93482	.06518	17
44	.81461	.87953	.93508	.06492	16
45	.81475	.87942	.93533	.06467	15
46	.81490	.87931	.93559	.06441	14
47	.81505	.87920	.93584	.06416	13
48	.81519	.87909	.93610	.06390	12
49	.81534	.87898	.93636	.06364	11
50	.81549	.87887	.93661	.06339	**10**
51	.81563	.87877	.93687	.06313	9
52	.81578	.87866	.93712	.06288	8
53	.81592	.87855	.93738	.06262	7
54	.81607	.87844	.93763	.06237	6
55	.81622	.87833	.93789	.06211	5
56	.81636	.87822	.93814	.06186	4
57	.81651	.87811	.93840	.06160	3
58	.81665	.87800	.93865	.06135	2
59	.81680	.87789	.93891	.06109	1
60	.81694	.87778	.93916	.06084	**0**
′	9 cos	9 sin	9 cot	10 tan	′

49°

41° 42°

′	9 sin	9 cos	9 tan	10 cot	′	′	9 sin	9 cos	9 tan	10 cot	′
0	.81694	.87778	.93916	.06084	**60**	**0**	.82551	.87107	.95444	.04556	**60**
1	.81709	.87767	.93942	.06058	59	1	.82565	.87096	.95469	.04531	59
2	.81723	.87756	.93967	.06033	58	2	.82579	.87085	.95495	.04505	58
3	.81738	.87745	.93993	.06007	57	3	.82593	.87073	.95520	.04480	57
4	.81752	.87734	.94018	.05982	56	4	.82607	.87062	.95545	.04455	56
5	.81767	.87723	.94044	.05956	55	5	.82621	.87050	.95571	.04429	55
6	.81781	.87712	.94069	.05931	54	6	.82635	.87039	.95596	.04404	54
7	.81796	.87701	.94095	.05905	53	7	.82649	.87028	.95622	.04378	53
8	.81810	.87690	.94120	.05880	52	8	.82663	.87016	.95647	.04353	52
9	.81825	.87679	.94146	.05854	51	9	.82677	.87005	.95672	.04328	51
10	.81839	.87668	.94171	.05829	**50**	**10**	.82691	.86993	.95698	.04302	**50**
11	.81854	.87657	.94197	.05803	49	11	.82705	.86982	.95723	.04277	49
12	.81868	.87646	.94222	.05778	48	12	.82719	.86970	.95748	.04252	48
13	.81882	.87635	.94248	.05752	47	13	.82733	.86959	.95774	.04226	47
14	.81897	.87624	.94273	.05727	46	14	.82747	.86947	.95799	.04201	46
15	.81911	.87613	.94299	.05701	45	15	.82761	.86936	.95825	.04175	45
16	.81926	.87601	.94324	.05676	44	16	.82775	.86924	.95850	.04150	44
17	.81940	.87590	.94350	.05650	43	17	.82788	.86913	.95875	.04125	43
18	.81955	.87579	.94375	.05625	42	18	.82802	.86902	.95901	.04099	42
19	.81969	.87568	.94401	.05599	41	19	.82816	.86890	.95926	.04074	41
20	.81983	.87557	.94426	.05574	**40**	**20**	.82830	.86879	.95952	.04048	**40**
21	.81998	.87546	.94452	.05548	39	21	.82844	.86867	.95977	.04023	39
22	.82012	.87535	.94477	.05523	38	22	.82858	.86855	.96002	.03998	38
23	.82026	.87524	.94503	.05497	37	23	.82872	.86844	.96028	.03972	37
24	.82041	.87513	.94528	.05472	36	24	.82885	.86832	.96053	.03947	36
25	.82055	.87501	.94554	.05446	35	25	.82899	.86821	.96078	.03922	35
26	.82069	.87490	.94579	.05421	34	26	.82913	.86809	.96104	.03896	34
27	.82084	.87479	.94604	.05396	33	27	.82927	.86798	.96129	.03871	33
28	.82098	.87468	.94630	.05370	32	28	.82941	.86786	.96155	.03845	32
29	.82112	.87457	.94655	.05345	31	29	.82955	.86775	.96180	.03820	31
30	.82126	.87446	.94681	.05319	**30**	**30**	.82968	.86763	.96205	.03795	**30**
31	.82141	.87434	.94706	.05294	29	31	.82982	.86752	.96231	.03769	29
32	.82155	.87423	.94732	.05268	28	32	.82996	.86740	.96256	.03744	28
33	.82169	.87412	.94757	.05243	27	33	.83010	.86728	.96281	.03719	27
34	.82184	.87401	.94783	.05217	26	34	.83023	.86717	.96307	.03693	26
35	.82198	.87390	.94808	.05192	25	35	.83037	.86705	.96332	.03668	25
36	.82212	.87378	.94834	.05166	24	36	.83051	.86694	.96357	.03643	24
37	.82226	.87367	.94859	.05141	23	37	.83065	.86682	.96383	.03617	23
38	.82240	.87356	.94884	.05116	22	38	.83078	.86670	.96408	.03592	22
39	.82255	.87345	.94910	.05090	21	39	.83092	.86659	.96433	.03567	21
40	.82269	.87334	.94935	.05065	**20**	**40**	.83106	.86647	.96459	.03541	**20**
41	.82283	.87322	.94961	.05039	19	41	.83120	.86635	.96484	.03516	19
42	.82297	.87311	.94986	.05014	18	42	.83133	.86624	.96510	.03490	18
43	.82311	.87300	.95012	.04988	17	43	.83147	.86612	.96535	.03465	17
44	.82326	.87288	.95037	.04963	16	44	.83161	.86600	.96560	.03440	16
45	.82340	.87277	.95062	.04938	15	45	.83174	.86589	.96586	.03414	15
46	.82354	.87266	.95088	.04912	14	46	.83188	.86577	.96611	.03389	14
47	.82368	.87255	.95113	.04887	13	47	.83202	.86565	.96636	.03364	13
48	.82382	.87243	.95139	.04861	12	48	.83215	.86554	.96662	.03338	12
49	.82396	.87232	.95164	.04836	11	49	.83229	.86542	.96687	.03313	11
50	.82410	.87221	.95190	.04810	**10**	**50**	.83242	.86530	.96712	.03288	**10**
51	.82424	.87209	.95215	.04785	9	51	.83256	.86518	.96738	.03262	9
52	.82439	.87198	.95240	.04760	8	52	.83270	.86507	.96763	.03237	8
53	.82453	.87187	.95266	.04734	7	53	.83283	.86495	.96788	.03212	7
54	.82467	.87175	.95291	.04709	6	54	.83297	.86483	.96814	.03186	6
55	.82481	.87164	.95317	.04683	5	55	.83310	.86472	.96839	.03161	5
56	.82495	.87153	.95342	.04658	4	56	.83324	.86460	.96864	.03136	4
57	.82509	.87141	.95368	.04632	3	57	.83338	.86448	.96890	.03110	3
58	.82523	.87130	.95393	.04607	2	58	.83351	.86436	.96915	.03085	2
59	.82537	.87119	.95418	.04582	1	59	.83365	.86425	.96940	.03060	1
60	.82551	.87107	.95444	.04556	**0**	**60**	.83378	.86413	.96966	.03034	**0**
′	9 cos	9 sin	9 cot	10 tan	′	′	9 cos	9 sin	9 cot	10 tan	′

48° 47°

43°

′	9 sin	9 cos	9 tan	10 cot	′
0	.83378	.86413	.96966	.03034	**60**
1	.83392	.86401	.96991	.03009	59
2	.83405	.86389	.97016	.02984	58
3	.83419	.86377	.97042	.02958	57
4	.83432	.86366	.97067	.02933	56
5	.83446	.86354	.97092	.02908	55
6	.83459	.86342	.97118	.02882	54
7	.83473	.86330	.97143	.02857	53
8	.83486	.86318	.97168	.02832	52
9	.83500	.86306	.97193	.02807	51
10	.83513	.86295	.97219	.02781	**50**
11	.83527	.86283	.97244	.02756	49
12	.83540	.86271	.97269	.02731	48
13	.83554	.86259	.97295	.02705	47
14	.83567	.86247	.97320	.02680	46
15	.83581	.86235	.97345	.02655	45
16	.83594	.86223	.97371	.02629	44
17	.83608	.86211	.97396	.02604	43
18	.83621	.86200	.97421	.02579	42
19	.83634	.86188	.97447	.02553	41
20	.83648	.86176	.97472	.02528	**40**
21	.83661	.86164	.97497	.02503	39
22	.83674	.86152	.97523	.02477	38
23	.83688	.86140	.97548	.02452	37
24	.83701	.86128	.97573	.02427	36
25	.83715	.86116	.97598	.02402	35
26	.83728	.86104	.97624	.02376	34
27	.83741	.86092	.97649	.02351	33
28	.83755	.86080	.97674	.02326	32
29	.83768	.86068	.97700	.02300	31
30	.83781	.86056	.97725	.02275	**30**
31	.83795	.86044	.97750	.02250	29
32	.83808	.86032	.97776	.02224	28
33	.83821	.86020	.97801	.02199	27
34	.83834	.86008	.97826	.02174	26
35	.83848	.85996	.97851	.02149	25
36	.83861	.85984	.97877	.02123	24
37	.83874	.85972	.97902	.02098	23
38	.83887	.85960	.97927	.02073	22
39	.83901	.85948	.97953	.02047	21
40	.83914	.85936	.97978	.02022	**20**
41	.83927	.85924	.98003	.01997	19
42	.83940	.85912	.98029	.01971	18
43	.83954	.85900	.98054	.01946	17
44	.83967	.85888	.98079	.01921	16
45	.83980	.85876	.98104	.01896	15
46	.83993	.85864	.98130	.01870	14
47	.84006	.85851	.98155	.01845	13
48	.84020	.85839	.98180	.01820	12
49	.84033	.85827	.98206	.01794	11
50	.84046	.85815	.98231	.01769	**10**
51	.84059	.85803	.98256	.01744	9
52	.84072	.85791	.98281	.01719	8
53	.84085	.85779	.98307	.01693	7
54	.84098	.85766	.98332	.01668	6
55	.84112	.85754	.98357	.01643	5
56	.84125	.85742	.98383	.01617	4
57	.84138	.85730	.98408	.01592	3
58	.84151	.85718	.98433	.01567	2
59	.84164	.85706	.98458	.01542	1
60	.84177	.85693	.98484	.01516	**0**
′	9 cos	9 sin	9 cot	10 tan	′

46°

44°

′	9 sin	9 cos	9 tan	10 cot	′
0	.84177	.85693	.98484	.01516	**60**
1	.84190	.85681	.98509	.01491	59
2	.84203	.85669	.98534	.01466	58
3	.84216	.85657	.98560	.01440	57
4	.84229	.85645	.98585	.01415	56
5	.84241	.85632	.98610	.01390	55
6	.84255	.85620	.98635	.01365	54
7	.84269	.85608	.98661	.01339	53
8	.84282	.85596	.98686	.01314	52
9	.84295	.85583	.98711	.01289	51
10	.84308	.85571	.98737	.01263	**50**
11	.84321	.85559	.98762	.01238	49
12	.84334	.85547	.98787	.01213	48
13	.84347	.85534	.98812	.01188	47
14	.84360	.85522	.98838	.01162	46
15	.84373	.85510	.98863	.01137	45
16	.84385	.85497	.98888	.01112	44
17	.84398	.85485	.98913	.01087	43
18	.84411	.85473	.98939	.01061	42
19	.84424	.85460	.98964	.01036	41
20	.84437	.85448	.98989	.01011	**40**
21	.84450	.85436	.99015	.00985	39
22	.84463	.85423	.99040	.00960	38
23	.84476	.85411	.99065	.00935	37
24	.84489	.85399	.99090	.00910	36
25	.84502	.85386	.99116	.00884	35
26	.84515	.85374	.99141	.00859	34
27	.84528	.85361	.99166	.00834	33
28	.84540	.85349	.99191	.00809	32
29	.84553	.85337	.99217	.00783	31
30	.84566	.85324	.99242	.00758	**30**
31	.84579	.85312	.99267	.00733	29
32	.84592	.85299	.99293	.00707	28
33	.84605	.85287	.99318	.00682	27
34	.84618	.85274	.99343	.00657	26
35	.84630	.85262	.99368	.00632	25
36	.84643	.85250	.99394	.00606	24
37	.84656	.85237	.99419	.00581	23
38	.84669	.85225	.99444	.00556	22
39	.84682	.85212	.99469	.00531	21
40	.84694	.85200	.99495	.00505	**20**
41	.84707	.85187	.99520	.00480	19
42	.84720	.85175	.99545	.00455	18
43	.84733	.85162	.99570	.00430	17
44	.84745	.85150	.99596	.00404	16
45	.84758	.85137	.99621	.00379	15
46	.84771	.85125	.99646	.00354	14
47	.84784	.85112	.99672	.00328	13
48	.84796	.85100	.99697	.00303	12
49	.84809	.85087	.99722	.00278	11
50	.84822	.85074	.99747	.00253	**10**
51	.84835	.85062	.99773	.00227	9
52	.84847	.85049	.99798	.00202	8
53	.84860	.85037	.99823	.00177	7
54	.84873	.85024	.99848	.00152	6
55	.84885	.85012	.99874	.00126	5
56	.84898	.84999	.99899	.00101	4
57	.84911	.84986	.99924	.00076	3
58	.84923	.84974	.99949	.00051	2
59	.84936	.84961	*.99975	.00025	1
60	.84949	.84949	.00000	.00000	**0**
′	9 cos	9 sin	10 cot	10 tan	′

45°

TABLE III.

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

AT INTERVALS

Of 5′ from 0° to 3° and 87° to 90°,
Of 10′ from 3° to 87°.

0° to 2°

	sin	cos	tan	cot	
0	.000000	1.00000	.00000	∞	**90°**
5′	1454	1.00000	145	687.55	55′
10′	2909	1.00000	291	343.77	50′
15′	4363	0.99999	436	229.18	45′
20′	5818	998	582	171.89	40′
25′	7272	997	727	137.51	35′
30′	.008727	996	.00873	114.59	30′
35′	.010181	995	.01018	98.218	25′
40′	1635	993	164	85.940	20′
45′	3090	991	309	76.390	15′
50′	4544	989	455	68.750	10′
55′	5998	987	600	62.499	5′
1°	.017452	.99985	.01746	57.290	**89°**
5′	.01891	982	.01891	52.882	55′
10′	.02036	979	.02036	49.104	50′
15′	181	976	182	45.829	45′
20′	327	973	328	42.964	40′
25′	472	969	473	40.436	35′
30′	618	966	619	38.188	30′
35′	763	962	764	36.178	25′
40′	.02908	958	.02910	34.368	20′
45′	.03054	953	.03055	32.730	15′
50′	199	949	201	31.242	10′
55′	345	944	346	29.882	5′
2°	.03490	.99939	.03492	28.636	**88°**
	cos	sin	cot	tan	

88° to 90°

2° to 5°

	sin	cos	tan	cot	
2°	.03490	.99939	.03492	28.636	**88°**
5′	635	934	638	27.490	55′
10′	781	929	783	26.432	50′
15′	.03926	923	.03929	25.452	45′
20′	.04071	917	.04075	24.542	40′
25′	217	911	220	23.695	35′
30′	362	905	366	22.904	30′
35′	507	898	512	22.164	25′
40′	653	892	658	21.470	20′
45′	798	885	803	20.819	15′
50′	.04943	878	949	20.206	10′
55′	.05088	870	.05095	19.627	5′
3°	.05234	.99863	.05241	19.081	**87°**
10′	524	847	533	18.075	50′
20′	.05814	831	.05824	17.169	40′
30′	.06105	813	.06116	16.350	30′
40′	395	795	408	15.605	20′
50′	685	776	700	14.924	10′
4°	.06976	.99756	.06993	14.301	**86°**
10′	.07266	736	.07285	13.727	50′
20′	556	714	578	13.197	40′
30′	.07846	692	.07870	12.706	30′
40′	.08136	668	.08163	12.251	20′
50′	426	644	456	11.826	10′
5°	.08716	.99619	.08749	11.430	**85°**
	cos	sin	cot	tan	

85° to 88°

5° to 15°

	sin	cos	tan	cot	
5°	.08716	.99619	.08749	11.430	**85°**
10′	.09005	.99594	.09042	11.059	50′
20′	295	567	335	10.712	40′
30′	585	540	629	.385	30′
40′	.09874	511	.09923	10.078	20′
50′	.10164	482	.10216	9.7882	10′
6°	.10453	.99452	.10510	9.5144	**84°**
10′	.10742	421	.10805	.2553	50′
20′	.11031	390	.11099	9.0098	40′
30′	320	357	394	8.7769	30′
40′	609	324	688	.5555	20′
50′	.11898	290	.11983	.3450	10′
7°	.12187	.99255	.12278	8.1443	**83°**
10′	476	219	574	7.9530	50′
20′	.12764	182	.12869	.7704	40′
30′	.13053	144	.13165	.5958	30′
40′	341	106	461	.4287	20′
50′	629	067	.13758	.2687	10′
8°	.13917	.99027	.14054	7.1154	**82°**
10′	.14205	.98986	351	6.9682	50′
20′	493	944	648	.8269	40′
30′	.14781	902	.14945	.6912	30′
40′	.15069	858	.15243	.5606	20′
50′	356	814	540	.4348	10′
9°	.15643	.98769	.15838	6.3138	**81°**
10′	.15931	723	.16137	.1970	50′
20′	.16218	676	435	6.0844	40′
30′	505	629	.16734	5.9758	30′
40′	.16792	580	.17033	.8708	20′
50′	.17078	531	333	.7694	10′
10°	.17365	.98481	.17633	5.6713	**80°**
10′	651	430	.17933	.5764	50′
20′	.17937	378	.18233	.4845	40′
30′	.18224	325	534	.3955	30′
40′	509	272	.18835	.3093	20′
50′	.18795	218	.19136	.2257	10′
11°	.19081	.98163	.19438	5.1446	**79°**
10′	366	107	.19740	5.0658	50′
20′	652	.98050	.20042	4.9894	40′
30′	.19937	.97992	345	.9152	30′
40′	.20222	934	648	.8430	20′
50′	507	875	.20952	.7729	10′
12°	.20791	.97815	.21256	4.7046	**78°**
10′	.21076	754	560	.6382	50′
20′	360	692	.21864	.5736	40′
30′	644	630	.22169	.5107	30′
40′	.21928	566	475	.4494	20′
50′	.22212	502	.22781	.3897	10′
13°	.22495	.97437	.23087	4.3315	**77°**
10′	.22778	371	393	.2747	50′
20′	.23062	304	.23700	.2193	40′
30′	345	237	.24008	.1653	30′
40′	627	169	316	.1126	20′
50′	.23910	100	624	.0611	10′
14°	.24192	.97030	.24933	4.0108	**76°**
10′	474	.96959	.25242	3.9617	50′
20′	.24756	887	552	.9136	40′
30′	.25038	815	.25862	.8667	30′
40′	320	742	.26172	.8208	20′
50′	601	667	483	.7760	10′
15°	.25882	.96593	.26795	3.7321	**75°**
	cos	sin	cot	tan	

75° to 85°

15° to 25°

	sin	cos	tan	cot	
15°	.25882	.96593	.26795	3.7321	**75°**
10′	.26163	517	.27107	.6891	50′
20′	443	440	419	.6470	40′
30′	.26724	363	.27732	.6059	30′
40′	.27004	285	.28046	.5656	20′
50′	284	206	360	.5261	10′
16°	.27564	.96126	.28675	3.4874	**74°**
10′	.27843	.96046	.28990	.4495	50′
20′	.28123	.95964	.29305	.4124	40′
30′	402	882	621	.3759	30′
40′	680	799	.29938	.3402	20′
50′	.28959	715	.30255	.3052	10′
17°	.29237	.95630	.30573	3.2709	**73°**
10′	515	545	.30891	.2371	50′
20′	.29793	459	.31210	.2041	40′
30′	.30071	372	530	.1716	30′
40′	348	284	.31850	.1397	20′
50′	625	195	.32171	.1084	10′
18°	.30902	.95106	.32492	3.0777	**72°**
10′	.31178	.95015	.32814	.0475	50′
20′	454	.94924	.33136	3.0178	40′
30′	.31730	832	460	2.9887	30′
40′	.32006	740	.33783	.9600	20′
50′	282	646	.34108	.9319	10′
19°	.32557	.94552	.34433	2.9042	**71°**
10′	.32832	457	.34758	.8770	50′
20′	.33106	361	.35085	.8502	40′
30′	381	264	412	.8239	30′
40′	655	167	.35740	.7980	20′
50′	.33929	.94068	.36068	.7725	10′
20°	.34202	.93969	.36397	2.7475	**70°**
10′	475	869	.36727	.7228	50′
20′	.34748	769	.37057	.6985	40′
30′	.35021	667	388	.6746	30′
40′	293	565	.37720	.6511	20′
50′	565	462	.38053	.6279	10′
21°	.35837	.93358	.38386	2.6051	**69°**
10′	.36108	253	.38721	.5826	50′
20′	379	148	.39055	.5605	40′
30′	650	.93042	391	.5386	30′
40′	.36921	.92935	.39727	.5172	20′
50′	.37191	827	.40065	.4960	10′
22°	.37461	.92718	.40403	2.4751	**68°**
10′	730	609	.40741	.4545	50′
20′	.37999	499	.41081	.4342	40′
30′	.38268	388	421	.4142	30′
40′	537	276	.41763	.3945	20′
50′	.38805	164	.42105	.3750	10′
23°	.39073	.92050	.42447	2.3559	**67°**
10′	341	.91936	.42791	.3369	50′
20′	608	822	.43136	.3183	40′
30′	.39875	706	481	.2998	30′
40′	.40141	590	.43828	.2817	20′
50′	408	472	.44175	.2637	10′
24°	.40674	.91355	.44523	2.2460	**66°**
10′	.40939	236	.44872	.2286	50′
20′	.41204	.91116	.45222	.2113	40′
30′	469	.90996	573	.1943	30′
40′	734	875	.45924	.1775	20′
50′	.41998	753	.46277	.1609	10′
25°	.42262	.90631	.46631	2.1445	**65°**
	cos	sin	cot	tan	

65° to 75°

25° to 35°

	sin	cos	tan	cot	
25°	.42262	.90631	.46631	2.1445	**65°**
10′	525	507	.46985	.1283	50′
20′	.42788	383	.47341	.1123	40′
30′	.43051	259	.47698	2.0965	30′
40′	313	133	.48055	809	20′
50′	575	.90007	414	655	10′
26°	.43837	.89879	.48773	2.0503	**64°**
10′	.44098	752	.49134	353	50′
20′	359	623	495	204	40′
30′	620	493	.49858	2.0057	30′
40′	.44880	363	.50222	1.9912	20′
50′	.45140	232	587	768	10′
27°	.45399	.89101	.50953	1.9626	**63°**
10′	658	.88968	.51319	486	50′
20′	.45917	835	.51688	347	40′
30′	.46175	701	.52057	210	30′
40′	433	566	427	.9074	20′
50′	690	431	.52798	1.8940	10′
28°	.46947	.88295	.53171	1.8807	**62°**
10′	.47204	158	545	676	50′
20′	460	.88020	.53920	546	40′
30′	716	.87882	.54296	418	30′
40′	.47971	743	.54673	291	20′
50′	.48226	603	.55051	165	10′
29°	.48481	.87462	.55431	1.8040	**61°**
10′	735	321	.55812	1.7917	50′
20′	.48989	178	.56194	796	40′
30′	.49242	.87036	577	675	30′
40′	495	.86892	.56962	556	20′
50′	.49748	748	.57348	437	10′
30°	.50000	.86603	.57735	1.7321	**60°**
10′	252	457	.58124	205	50′
20′	503	310	513	.7090	40′
30′	.50754	163	.58905	1.6977	30′
40′	.51004	.86015	.59297	864	20′
50′	254	.85866	.59691	753	10′
31°	.51504	.85717	.60086	1.6643	**59°**
10′	.51753	567	483	534	50′
20′	.52002	416	.60881	426	40′
30′	250	264	.61280	319	30′
40′	498	.85112	.61681	212	20′
50′	745	.84959	.62083	107	10′
32°	.52992	.84805	.62487	1.6003	**58°**
10′	.53238	650	.62892	1.5900	50′
20′	484	495	.63299	798	40′
30′	730	339	.63707	697	30′
40′	.53975	182	.64117	597	20′
50′	.54220	.84025	528	497	10′
33°	.54464	.83867	.64941	1.5399	**57°**
10′	708	708	.65355	301	50′
20′	.54951	549	.65771	204	40′
30′	.55194	389	.66189	108	30′
40′	436	228	.66608	.5013	20′
50′	678	.83066	.67028	1.4919	10′
34°	.55919	.82904	.67451	1.4826	**56°**
10′	.56160	741	.67875	733	50′
20′	401	577	.68301	641	40′
30′	641	413	.68728	550	30′
40′	.56880	248	.69157	460	20′
50′	.57119	.82082	.69588	370	10′
35°	.57358	.81915	.70021	1.4281	**55°**
	cos	sin	cot	tan	

55° to 65°

35° to 45°

	sin	cos	tan	cot	
35°	.57358	.81915	.70021	1.4281	**55°**
10′	596	748	455	193	50′
20′	.57833	580	.70891	106	40′
30′	.58070	412	.71329	.4019	30′
40′	307	242	.71769	1.3934	20′
50′	543	.81072	.72211	848	10′
36°	.58779	.80902	.72654	1.3764	**54°**
10′	.59014	730	.73100	680	50′
20′	248	558	547	597	40′
30′	482	386	.73996	514	30′
40′	716	212	.74447	432	20′
50′	.59949	.80038	.74900	351	10′
37°	.60182	.79864	.75355	1.3270	**53°**
10′	414	688	.75812	190	50′
20′	645	512	.76272	111	40′
30′	.60876	335	.76733	.3032	30′
40′	.61107	.79158	.77196	1.2954	20′
50′	337	.78980	.77661	876	10′
38°	.61566	.78801	.78129	1.2799	**52°**
10′	.61795	622	.78598	723	50′
20′	.62024	442	.79070	647	40′
30′	251	261	.79544	572	30′
40′	479	.78079	.80020	497	20′
50′	706	.77897	498	423	10′
39°	.62932	.77715	.80978	1.2349	**51°**
10′	.63158	531	.81461	276	50′
20′	383	347	.81946	203	40′
30′	608	.77162	.82434	131	30′
40′	.63832	.76977	.82923	.2059	20′
50′	.64056	791	.83415	1.1988	10′
40°	.64279	.76604	.83910	1.1918	**50°**
10′	501	417	.84407	847	50′
20′	723	229	.84906	778	40′
30′	.64945	.76041	.85408	708	30′
40′	.65166	.75851	.85912	640	20′
50′	386	661	.86419	571	10′
41°	.65606	.75471	.86929	1.1504	**49°**
10′	.65825	280	.87441	436	50′
20′	.66044	.75088	.87955	369	40′
30′	262	.74896	.88473	303	30′
40′	480	703	.88992	237	20′
50′	697	509	.89515	171	10′
42°	.66913	.74314	.90040	1.1106	**48°**
10′	.67129	.74120	.90569	1.1041	50′
20′	344	.73924	.91099	1.0977	40′
30′	559	728	.91633	913	30′
40′	773	531	.92170	850	20′
50′	.67987	333	.92709	786	10′
43°	.68200	.73135	.93252	1.0724	**47°**
10′	412	.72937	.93797	661	50′
20′	624	737	.94345	599	40′
30′	.68835	537	.94896	538	30′
40′	.69046	337	.95451	477	20′
50′	256	.72136	.96008	416	10′
44°	.69466	.71934	.96569	1.0355	**46°**
10′	675	732	.97133	295	50′
20′	.69883	529	.97700	235	40′
30′	.70091	325	.98270	176	30′
40′	298	.71121	.98843	117	20′
50′	505	.70916	.99420	058	10′
45°	.70711	.70711	1.00000	1.0000	**45°**
	cos	sin	cot	tan	

45° to 55°

COMPUTATION RULES

AND

LOGARITHMS.

WITH TABLES OF OTHER USEFUL FUNCTIONS.

BY

SILAS W. HOLMAN,

Professor of Physics at the Massachusetts Institute of Technology.

PRESS NOTICES.

"A very useful book for the practical computer." — *American Mathematical Monthly.*

"The aim of this book, as the author states in his preface, is to provide a method by which the student or engineer who has computations to make can economize time and labor through an intelligent command of his use of figures, and to decide at a glance what number of places is sufficient for the purpose in hand. One half of the time expended in computations is wasted through the use of an excessive number of places of figures, and through failure to employ logarithms. The author's purpose is to enable this waste to be avoided by following a few simple computation rules. . . . The rules are very simple and direct, and were evidently prepared with the view of their being easily retained in the memory, and to aid in fixing them in the mind the principal portions of the rules are printed in bold-face type. . . . The tables are very well arranged and are exceedingly convenient, as it would appear from an experiment with them. . . . The book is well arranged and well bound, and the letter press is excellent. The paper is extra heavy and well suited to the purpose." — *Railway Review.*

THE MACMILLAN COMPANY,

66 FIFTH AVENUE, NEW YORK.

ELEMENTARY ALGEBRA.

BY

H. S. HALL, B.A., and S. R. KNIGHT, B.A.,

REVISED AND ENTIRELY REWRITTEN FOR USE OF AMERICAN SCHOOLS BY

FRANK L. SEVENOAK, A.M., M.D.,

Professor of Mathematics and Natural Sciences, and Assistant Principal of Stevens School, Hoboken, N.J.

16mo. Half Leather. $1.10.

PRESS NOTICES.

"In the Elementary Algebra, we are treated to a good book made better. Hall and Knight's works have been and are extensively used in England, and not a little in America, and their popularity is well deserved. They supply a need in the way of text-books pitched a degree higher than the ordinary run of school literature. They are books that make students think, and hence this revision should be well received in this country." — *School Review.*

"We will not say that this is the best Elementary Algebra for school use that we have come across, but we can say that *we do not remember to have seen a better. . . . It is the outcome of a long experience of school-teaching, and so is a thoroughly practical book.* All others that we have in our eye are the works of men who have had considerable experience with senior and junior students at the universities, but have had little, if any, acquaintance with the poor creatures who are just stumbling over the threshold of algebra. . . . Buy or borrow the book for yourselves and judge." — *Academy.*

ALGEBRA FOR BEGINNERS.

BY

H. S. HALL, M.A., and S. R. KNIGHT.

REVISED BY

FRANK L. SEVENOAK, A.M., M.D.,

Assistant Principal and Professor of Mathematics and Natural Sciences, in the Stevens School, Hoboken, N.J.

16mo. Cloth. 60 cents.

"Although elementary, the work is so much superior to several popular American text-books for beginners that it is hoped that it may find a place in our schools." — *The School Review.*

"A very complete treatise on the elements of algebra as taught in our schools. . . . The special aim of the book seems to be to bring out the principles as they are to be applied in scientific work. This edition is an improvement on the English one, which is itself a most excellent book." — *The Critic.*

"This work deserves high rank as a text-book on algebra. It is one of the simplest yet one of the most thorough works on elementary algebra." — *Notre Dame Scholastic.*

"A very concise and complete treatise." — *Journal of Education.*

THE MACMILLAN COMPANY,
66 FIFTH AVENUE, NEW YORK.

This book should be returned to the Library on or before the last date stamped below.

A fine of five cents a day is incurred by retaining it beyond the specified time.

Please return promptly.